PROGRAMS FOR STATISTICAL EVALUATION OF MEASURED CLUTTER DATA

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INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1989

TO

THE MEMORY OF

MY LATE FATHER DR. P.N. JAYASWAL, FRCS

AND

TO

MY MOTHER, SUBHASH, GAUTAM AND ALOKA

CERTIFICATE

It is certified that the work entitled 'Programs for Statistical Evaluation of Measured Clutter Data' by Major Rajiv Jayaswal has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

April ,1989

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R. Jayaswal

ABSTRACT

A fluctuating extended radar target is a general model for all classes of clutter-producing media. The complex reflectivity of a target can be estimated. Based on this data, and under the assumption of WSSUS, various properties of interest can be investigated. Using common tools of analysis, three programs have been developed for evaluating:

1. The probability distribution of clutter samples obtained from a single range cell.

2. Correlation between the clutter data of two range cells.

3. Clutter Power Spectral Density of a single range cell.

These programs (in FORTRAN IV) were run on mini-computer PDP-11/24 with generated test data and produced satisfactory results. Some limitations of the methods used are discussed and a few suggestions for improvement given.

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CHAPTER 1

INTRODUCTION

1.1 PREAMBLE

When a beam of electromagnetic energy is transmitted from a directional radar antenna, the signal received by the radar would comprise of

a) reflection from the desired target (s), if any,

b) unwanted reflection from other than the desired targets in the path of the beam, termed CLUTTER,

c) external noise generated by electrical disturbances,

d) thermal noise at the receiver input.

The difference between clutter and noise is that an increase in the transmitted power results in an increase in the clutter power as well as the desired signal level but not in the noise level.

The classical problem in radar signal processing is to detect the desired target signal which is embedded in clutter. The obvious method of doing this is to enhance the target signal relative to the clutter by exploiting the differences in spatial, temporal and spectral properties of the target as compared to the clutter source.

For example the traditional method of clutter suppression is the MTI (Moving Target Indication) filter. This works on the

assumption that the target of interest is moving radially to the radar while the clutter source is not. This assumption is not always correct.

More recently, various adaptive clutter suppression filters have been designed or theoretically formulated. Their principle is to adapt to the prevalent clutter conditions and in subsequent processing enhance the signal-to-clutter ratio, one way being to subtract an estimate of the clutter out of the total received signal, the desired target signal remaining basically unaffected.

A study of the statistical properties of land, sea and weather clutter is of interest. Extensive studies of clutter [illegible] collection and analysis of data have already been carried out in various countries. However, the studies on land clutter do not relate to the Indian subcontinent and are, therefore, of no particular use to us [1].

It would, therefore, be gainful to measure clutter data from various types of terrain in this country, under different climatic conditions, and to carry out studies on the data. Due to the random nature of the clutter and the large masses of the data involved, the study would have to be based on statistical evaluation.

The time varying nature of clutter, its degree of spatial correlation and spectral content are important aspects of such a statistical evaluation. Some methods and programs for

carrying out this evaluation are considered in this thesis. However, these, at most form a preliminary investigation into the clutter data and more detailed work in this direction is recommended.

A better understanding of clutter properties would in future influence the design of radar receivers/adaptive clutter suppression systems.

1.2 RADAR CLUTTER DATA MEASUREMENT

1.2.1 Brief Background:

A clutter signal consists of unwanted radar returns. In experimental studies of clutter producing targets, however, the clutter is a source of information. Such a study can be viewed as consisting of two interlinked parts:

a) The measurement part

b) The statistical analysis part

A vehicle mounted clutter measurement system based on a CW radar has been designed and setup at IIT [1]. The specifications are given at Table 1.1. It would constitute the source of measured clutter data.

In order to analyse this statistical data, off-line, the implementation of relevant statistical methods on a minicomputer (PDP-11/24) using FORTRAN language, was envisaged. This forms the subject of the present work.

The result was the development of 3 programs for statistical processing of clutter data. These relate, respectively, to

a) Probability distribution

b) Spatial correlation

c) Power spectral density

A consolidated list of environmental and system parameters to be recorded 'on-site' is suggested in Table 1.2.

1.2.2 Model:

A general model for a clutter source is a randomly fluctuating, range-spread target. Under the assumption of WSSUS (wide - sense stationarity and uncorrelated scattering), and using complex envelope notation as applied to narrow-band bandpass signals, the complex envelope of the received (clutter) signal is given by [2]

$$\tilde{s}(t) = \int_{-\infty}^{\infty} \sqrt{E_t}\, \tilde{f}(t-\lambda)\, \tilde{b}(t - \tfrac{\lambda}{2}, \lambda)\, d\lambda \qquad (1.1)$$

where,

E_t is the energy of the transmitted waveform

$\tilde{f}(t)$ is the complex envelope of the transmitted signal normalised to unit energy

λ is the two-way range-delay in time units

$\tilde{b}(t,\lambda)$ is the complex reflectivity of a $d\lambda$ wide strip located at range-delay $\frac{\lambda}{2}$.

It should be noted that $\tilde{b}$ includes path losses, antenna gains, effect of polarisation, etc.

The clutter source is equivalent to a random, linear, time-varying filter with input $\sqrt{E_t}\ \tilde{f}(t)$ (complex envelope of the transmitted signal), impulse response $\tilde{b}(t - \frac{\lambda}{2}, \lambda) = \tilde{b}(t, \lambda)$, and output $\tilde{s}(t)$ (complex envelope of the received clutter signal).

The measurement system is designed to provide estimates of the samples of the complex reflectivity (or impulse response) $\tilde{b}$ as a function of both discrete range and discrete time.

Integer variables K and n will be used to denote the discrete range and the discrete time respectively.

For a particular set of parameters in Table 1.2, the data would consist of the equispaced-in-time sequence of sample estimates $\tilde{b}_K^{\ n}$. Each $\tilde{b}_K^{\ n}$ consists of the in-phase and quadrature components, i.e. two real numbers.

If the beam direction is kept fixed and assuming that the illuminated patch consists of r range cells given by

$$K = K_1, K_2, K_3, \ldots, K_r \ ; \ 0 \leq K \leq 32767 \qquad (1.2)$$

the recorded data would consist of N (e.g. 1024) windows, the nth window comprising the sequence

$$\tilde{b}_{K_1}^{\ n}, \tilde{b}_{K_2}^{\ n}, \ldots \tilde{b}_{K_r}^{\ n} \ ; \ n = 1,2,\ldots,N \ ;$$

$$\tilde{b}_{K_i}^{\ n} = (b_{I,K_i}^{\ n}, b_{Q,K_i}^{\ n}) \qquad (1.3)$$

where I and Q denote in-phase and quadrature-phase respectively.

It may be noted that the window index n is the same as the discrete time index for data samples pertaining to a given range cell K_i.

The data will be recorded on a storage device (Mag tape).

1.2.3 Preprocessing:

A re-arrangement of the above database is required prior to subjecting it to the envisaged statistical processing.

The original database can be thought of as an Nxr matrix (e.g. N = 512, r = 173) whose N rows are the N windows. Let the matrix be denoted by $[\vec{b}_{n,K}]$.

The data is now required range-cell-wise. This can be achieved by a transposition of the above time-range matrix, yielding

$$[\ \vec{b}_{n,K}\ ]^T = [\ \vec{b}_{K,n}] \tag{1.4}$$

This preprocessing manipulation has been presumed. However, it can be included as the first step in the statistical processing.

The methods in Chapters 2&3 process N-point data sequences pertaining to individual cells, i.e. rows of the transposed matrix.

1.3 AN OUTLINE OF THE THESIS

The statistical processing/analysis scheme comprises three parts, one for each of the properties to be investigated.

To observe whether the time-variations of the clutter in a given range cell match a known distribution, Chapter 2 describes

qualitative tests for Rayleigh, Weibull and Lognormal models. Two examples illustrate the use of this method.

The first part of Chapter 3 deals with the estimation of clutter power spectral density. A scattering function description of a doubly-spread radar target, based on the assumption of WSSUS, is given. An approximate model with a tapped-delay-line structure is described. The FFT algorithm used in the program is explained. One example is given.

The second part of Chapter 3 deals with the estimation of correlation between the clutter returns from two range cells. Two methods (Direct and FFT) for the estimation are outlined and some illustrative examples given.

Chapter 4 summarises the programs developed and gives few suggestions for improvement.

All examples are based on generated data.

TABLE 1.1

MEASUREMENT SYSTEM SPECIFICATIONS

System	:	Truck-mounted S and X Band CW radar
Transmitted Power	:	400W
IF	:	70 MHz
Modulating Signal	:	PRBS generated by 15-stage SR
Type of modulation	:	Biphase
Clock rates	:	1.25, 2.5, 5, 10 MHz
Antenna	:	Parabolic dish, dia 1 m at S-band $\frac{1}{2}$ m at X-band
Polarization	:	Horizontal/Vertical (switchable)
Sidelobe level	:	-20 dB max.
Antenna gain	:	25 dB at S-band 33 dB at X-band
Scan rate	:	10 rpm, in Azimuth, over 360°
Angle of depression	:	0.5° to 17.5° (below horizontal)

Max. unambiguous range corresponds to the 32767th range cell

Positional accuracy	:	Azimuth $\pm 0.5^{\circ}$ Elevation (depression) $\pm 0.25^{\circ}$
System output	:	Samples of complex reflectivity (in- and quadrature-phase components) of 15 adjacent range-cells are simultaneously estimated.

(The measurement system was designed at A.C.E.S., IITK)

TABLE 1.2

ENVIRONMENTAL AND SYSTEM PARAMETERS

	Parameter	Typical example
1.	Date	150688
2.	Place	KANPUR
3.	Terrain	CITY
4.	Weather	CLEAR
5.	Temperature (^{o}C)	28.5
6.	Pressure (mm)	180
7.	Humidity (%)	80.5
8.	Wind Velocity (km/h)	0
9.	Carrier frequency (GHz)	3.0
10.	Clock-rate (MHz)	10.0
11.	Depression angle (^{o})	15.5
12.	Azimuthal angle (^{o})	250
13.	Polarisation (V/H)	H
14.	Time	1615
15.	Samples/cell	1024

CHAPTER 2

STATISTICAL DISTRIBUTION OF THE CLUTTER SAMPLES FROM ONE RANGE CELL: A QUALITATIVE EVALUATION

2.1 PURPOSE

For any class of clutter sources, a general model is a randomly fluctuating, range-extended (doubly spread) radar target, e.g. a thickly vegetated hillside in windy weather.

The prevailing weather contributes its own factors (such as rain, dust, snow, hail, wind) to the random nature of the received clutter signal.

For instance, the spectral content of clutter from a wooded area would vary with the wind speed through that area.

It is of interest to study the clutter returns from various types of reflecting surfaces such as cultivated land, water stretches, deserts, forests, hills, snow-clad mountains, built-up areas, and also to relate the nature of the clutter variations to the general class of the source, its physical spread, as well as geographical and meteorological factors.

The extent or spread of the target (in the illuminated space) is obtained by the number of range cells which contribute to the clutter returns.

Estimates of the complex reflectivity of each range cell

in the illuminated patch as a function of (discrete) time, $\tilde{b}_K^{\,n}$, where

$\tilde{b}$ denotes complex reflectivity,

n denotes the discrete time index, $1 \leq n \leq N$,

K denotes the discrete range index,

can be obtained with the radar system described in Section 1.2.

For the Kth range cell, if a set of N observations are taken, the resulting data comprises of a set of N complex numbers

$$\tilde{b}_K^{\,1}\ ,\ \tilde{b}_K^{\,2}\ ,\ \tilde{b}_K^{\,3}, \ldots,\ \tilde{b}_K^{\,N}$$

representing N complex-reflectivity estimates equi-spaced-in-time.

The question that logically follows is that does the distribution of these sample values follow any of the known probability distributions?

Three statistical models have been suggested for the magnitude of the complex reflectivity, $|\tilde{b}_K^{\,n}|$, in the literature (e.g. [2],[3]) for all types of terrain clutter. These are the Rayleigh, Weibull and Lognormal distributions. Their density/distribution functions are given in Table 2.1.

The adequacy of these distributions, in relation to the Indian terrain-and-meteorological conditions, for modelling the time-varying clutter return, requires verification. One approach in carrying out such a check is to assume a particular statistical model and then to test the validity of the assumption with the data. This is the approach used in this work.

TABLE 2.1

RALEIGH, WEIBULL AND LOGNORMAL DISTRIBUTION FUNCTIONS

RAYLEIGH

$$p(r) = \begin{cases} 0, & r < 0 \\ \dfrac{r}{\sigma^2} e^{-r^2/2\sigma^2}, & r \geq 0 \end{cases}$$

$$F(r) = \begin{cases} 0, & r < 0 \\ 1 - e^{-r^2/2\sigma^2}, & r \geq 0 \end{cases}$$

$$-\infty < \sigma < \infty$$

WEIBULL

$$p(x) = \begin{cases} 0, & x < 0 \\ \dfrac{\eta}{\sigma} \left(\dfrac{x}{\sigma}\right)^{\eta-1} \exp\left[-\left(\dfrac{x}{\sigma}\right)^{\eta}\right], & x \geq 0 \end{cases}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-(x/\sigma)^{\eta}}, & x \geq 0 \end{cases}$$

$$\eta > 0, \quad \sigma > 0$$

LOGNORMAL

$$p(y) = \begin{cases} 0, & y < 0 \\ \dfrac{1}{\sqrt{2\pi}\,\sigma y} e^{-\dfrac{(\ln y-\mu)^2}{2\sigma^2}}, & y \geq 0 \end{cases}$$

$$F(Y) = \begin{cases} 0, & y < 0 \\ \displaystyle\int_0^Y \frac{1}{\sqrt{2\pi}\sigma y} e^{\dfrac{-(\ln y-\mu)^2}{2\sigma^2}} dy, & y \geq 0 \end{cases}$$

$$\sigma > 0, \quad -\infty < \mu < \infty$$

Symbols: p: probability density function (pdf), F: Cumulative Distribution Function (CDF); r,x,y: random variables (RV), Y: particular value of y.

2.2 TEST FOR RAYLEIGH AND WEIBULL DISTRIBUTIONS

The data relates to the random variable

$$x = b = |\tilde{b}| \tag{2.1}$$

for a given range-cell, whose distribution is to be studied. It suffices to test for a Weibull distribution only, the Rayleigh model being a special case of the Weibull model. The Rayleigh model shall be valid when the following two conditions are met:

a) Weibull model holds good

b) the Weibull parameter $\eta = 2$

A simple method to assess the validity of an assumed (in this case Weibull) distribution is to manipulate the measured statistical data in such a manner that, when plotted graphically, it:

a) plots into a straight line if the statistical model is valid ,

b) does not plot into a straight line in all other cases.

The cumulative Distribution Function of the Weibull distribution is

$$F(x) = 1 - \exp [- (x/\sigma)^{\eta}], \; x \geq 0, \; \eta > 0, \; \sigma > 0 \tag{2.2}$$

After algebraic manipulation this can be written as

$$\ln\ln \frac{1}{1-F(x)} = \eta \ln x - \eta \ln\sigma, \; x \geq 0, \; \eta > 0, \; \sigma > 0 \tag{2.3}$$

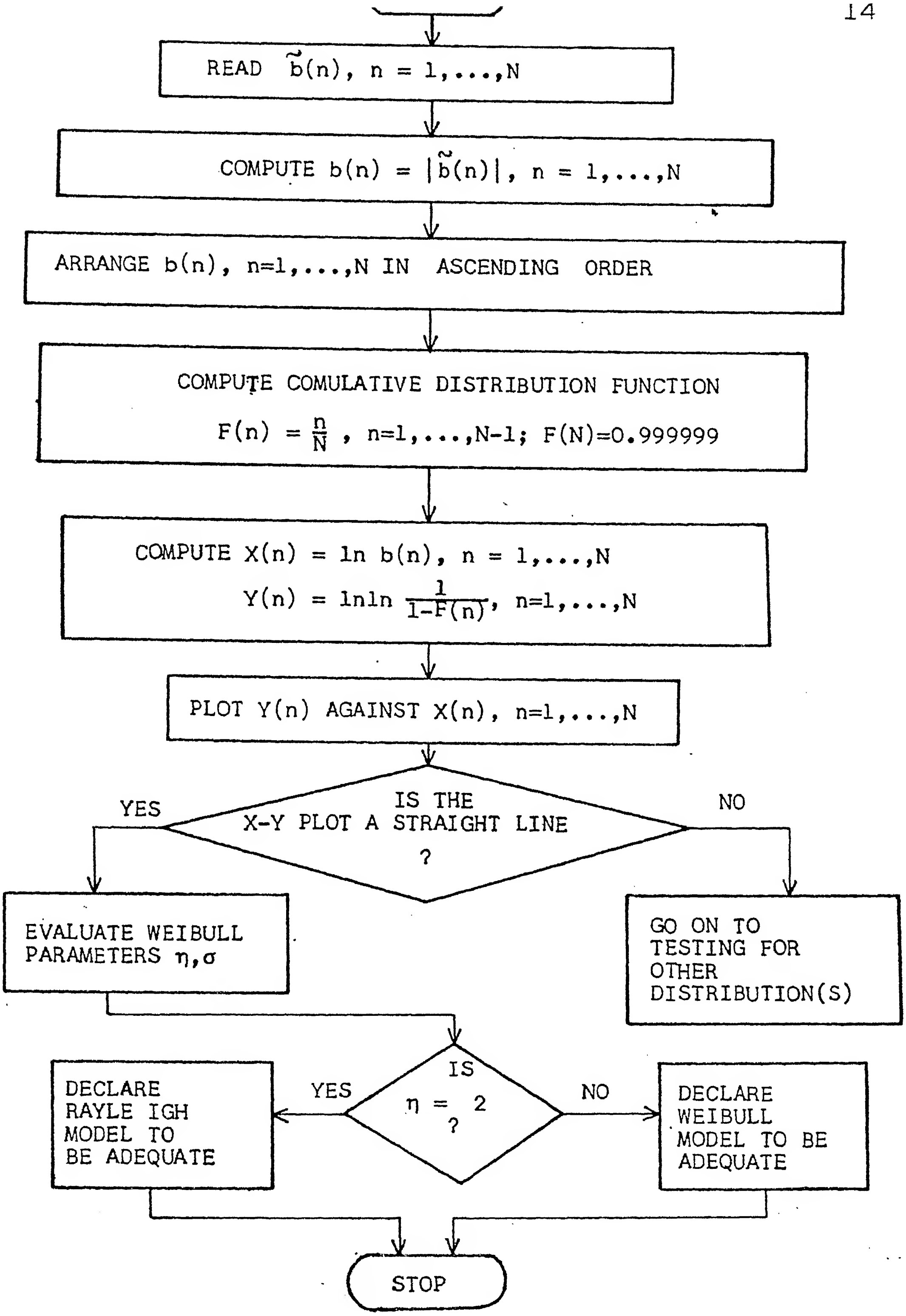


Fig. 2.1: SCHEME FOR TESTING THE VALIDITY OF RAYLEIGH AND WEIBULL MODELS

which is the equation of a straight line with slope η and y-axis intercept $(-\eta \ln \sigma)$.

The various steps in the complete test procedure are shown in the flow chart of Fig. 2.1. This forms the basis for the first half of Program No. 1, the second half consisting of a similar test for the lognormal distributional model.

2.3 TEST FOR LOGNORMAL DISTRIBUTION

The Cumulative Distribution Function for the Lognormal model is, from Table 2.1,

$$F(B) = \int_0^B \frac{1}{\sqrt{2\pi}\sigma b} \exp\left[-\frac{(\ln b-\mu)^2}{2\sigma^2}\right]db, \tag{2.4}$$

$$b \geq 0, \ \sigma > 0, \ -\infty < \mu < \infty$$

where,

the random variable $b = |\tilde{b}|$

B is a particular value of b

μ is the mean of lnb

σ^2 is the variance of lnb

With a change of variable $x' = \ln b$, eqn. (2.4) becomes

$$F(B) = \int_{-\infty}^{\ln B} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]dx, \tag{2.5}$$

$$-\infty < x < \infty, \ \sigma > 0, -\infty < \mu < \infty$$

which is a Normal (μ, σ^2) distribution.

By a second change of variable $z = \frac{x-\mu}{\sigma}$,

$$F(B) = \int_{-\infty}^{(\ln B-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} \exp[-\frac{z^2}{2}]dz, \quad \sigma>0, \tag{2.6}$$

$$-\infty<z<\infty, \quad -\infty<\mu<\infty$$

which is the error function in classical Detection and Estimation theory.

$$F(B) = \mathrm{erf}\left(\frac{\ln B-\mu}{\sigma}\right) = Q\left(\frac{\ln B-\mu}{\sigma}\right) \tag{2.7}$$

Since $z = \frac{\ln b-\mu}{\sigma}$, we put $Z = \frac{\ln B-\mu}{\sigma}$ (2.8)

where Z is a particular value of the RV z.

$$F(B) = \mathrm{erf}(Z) = Q(Z), \quad B>0, -\infty<Z<\infty \tag{2.9}$$

If the lognormal model is valid, a plot of $F(B(I))$ against $Q(Z(I))$, $I = 1,2,\ldots,N$ would be a straight line through the origin with unity slope.

On the face of it this test (for the lognormal model) seems more rigorous than that (in 2.2) for the Rayleigh and Weibull models. However, the calculation of $Q(Z) = Q(\frac{\ln B-\mu}{\sigma})$ requires the values of μ and σ, which are not previously known and have to be evaluated from the data itself. Thus μ and σ are themselves RVs, which would vary with each set of measured data. The introduction of the two RVs has to be offset against the greater ease in (visual) assessment from the graphical plot. The value of the error function integral

$$Q(Z) = \mathrm{erf}(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz \tag{2.10}$$

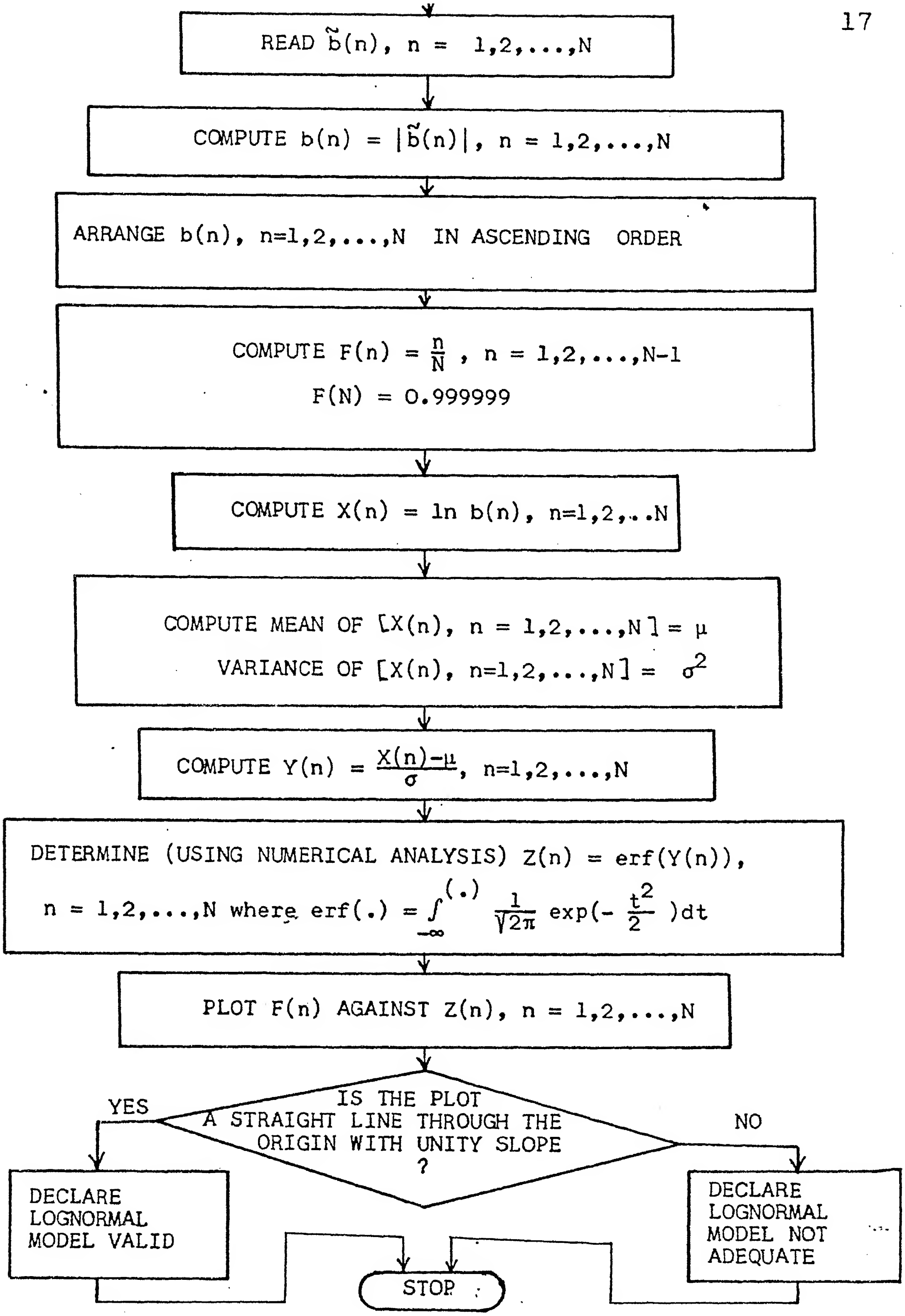


Fig. 2.2: SCHEME FOR TESTING THE VALIDITY OF LOGNORMAL MODEL

is evaluated (by a subroutine) using the Gaussian quadrature method of Numerical Analysis [5].

The complete procedure involved in this test is shown as the flowchart in Fig. 2.2. The first 5 steps are identical to those in Fig. 2.1, being, in the program, subroutines commonly shared.

2.4 DISCUSSION AND SUMMARY

1. Successive valuesof the measured parameter

$$\tilde{b}(n),\ n=1,\ldots,N \tag{2.11}$$

should be sufficiently separated in time as to be uncorrelated. In case this condition is not satisfied during the measurement phase, the entire period corresponding to n=1,...,N can be divided into intervals equal to the requisite separation (except for the last interval which may be of a smaller length) and the samples averaged out in each of these intervals. For this reason a linear time averaging facility has been provided in the menu-driven program and is optional. (The number of consecutive samples to be averaged is set by the user).

The number of data points after averaging, if any, should be sufficient for meaningful post-processing. This entails taking an adequate number of data points (N) to start with.

2. The testing is qualitative and, in each case, provides for one of the following decisions (from the graphical plots):

a) the model under test is adequate

b) the model under test appears to be adequate

c) the model under test is rejected as inadequate

d) the number of points are too few to make a decision.

As the number of data points is increased, the assessment becomes correspondingly easier upto a point, whereafter further increase would only cluster the plotted points together unless the scale is correspondingly expanded.

However, in the lognormal test, as the mean and variance of $\ln|\tilde{b}|$, (μ,σ^2), are estimated from the data itself, the accuracy of the estimates increases with N.

For example, the estimates of (μ,σ^2) were (.3,1.7) with N = 16 when the test was run with data from a simulated lognormal model with actual $(\mu,\sigma^2) = (0,1)$. Increase in N gave more accurate estimates for (μ,σ^2). For N = 1024, (μ,σ^2) were (.00278, 1.02), the plot was a straight line with slope = 1.0107 and the Y-axis intercept = -4.758×10^{-3}

3. The basic limitation of this model-fit test procedure is its qualitative nature. It is subjective and provides a preliminary insight into the time-varying behaviour of the data. It is, therefore, suggested that a second test, which is quantitative in nature, be included in the testing schemes, at a later stage, for corroboration, especially in cases where the decision is of the form 2(b) above.

4. The program which has been written incorporating the method outlined in this chapter is menu-driven and essentially consists of two parts:

a) the Rayleigh/Weibull model validity assessment

b) the Lognormal model validity assessment

The user is provided the facility to:

a) carry out one or both of these two tests

b) repeat one or both tests

without having to re-run the program.

5. As an illustration, the results of running the program with generated test data, for the following two cases, are shown:

a) Rayleigh distributed data

b) Lognormal distributed data

on subsequent pages. (The data was generated with the help of the program in Appendix A.4).

Example 1:

We consider a case where we are given a sequence of Rayleigh distributed clutter samples which we subject to the two tests for (a) the Rayleigh/Weibull (b) the Lognormal model respectively.

Table 2.2 shows N = 32 samples of the complex reflectivity pertaining to a range cell (say Gth cell), whose magnitudes follow a Rayleigh distribution with parameter $\sigma = 1.5$.

No time-averaging was done on the data.
The data was processed by the Rayleigh/Weibull test program segment and the resultant data is shown in Table 2.3 and its plot in Fig. 2.3. The straight line obtained indicates that the data is in general, Weibull distributed and the slope $\eta = 2.0$ indicates that it is in particular, Rayleigh distributed. The Rayleigh distribution parameter estimate obtained from the plot is

$$\sigma_E = 1.5000007$$

which compares favourably with the σ value of the generated data.

The same data was next processed by the lognormal test program segment and the resultant data is shown in Table 2.4 and is plotted in Fig. 2.4. The best straight line fit is indicated. The plot is not a straight line which appears to indicate that a lognormal model for the given data is not adequate. However, as pointed out in Section 2.4, the lognormal model test method is based on

TABLE 2.2
Cell data

no.	In-phase part	quadrature part
1	0.27911732E+01	0.85210212E-01
2	0.55312514E+01	0.67858332E+00
3	0.21003673E+01	0.95628780E+00
4	0.49009085E+00	0.24491093E+01
5	-0.92636120E+00	-0.18216825E+01
6	0.20585775E+01	0.66408443E+00
7	0.80691236E+00	0.16638813E+01
8	-0.17642403E+01	-0.80864424E+00
9	-0.63073218E-01	-0.16882457E+01
10	0.12802414E+01	0.12166116E+01
11	-0.13160012E+01	0.81901288E+00
12	-0.37892115E+00	0.15727392E+01
13	0.14483830E+00	-0.14162635E+01
14	0.39775985E+00	0.14313120E+01
15	-0.49155420E+00	-0.12096145E+01
16	0.13566177E+01	-0.14005318E+00
17	0.11907479E+01	0.72769538E-01
18	0.12204448E+01	0.26475656E+00
19	0.82700241E+00	0.69927490E+00
20	-0.66707754E+00	0.92172498E+00
21	0.88461250E+00	0.40643814E+00
22	0.12938941E+00	0.10201796E+01
23	-0.16734710E+00	-0.84559977E+00
24	0.33419365E+00	0.85520470E+00
25	-0.57129407E+00	-0.47860053E+00
26	-0.14214128E+00	-0.79188430E+00
27	0.57653236E+00	0.22334610E+00
28	0.59902543E+00	-0.32917646E+00
29	-0.46995527E+00	-0.25142889E-01
30	-0.50128603E+00	0.22171623E+00
31	0.24894603E+00	0.97264990E-01
32	0.31414589E+00	0.21569446E+00

TABLE 2.3

WEIBULL DISTRIBUTION TEST : OUTPUT DATA

	X	Y
1	-0.13194865E+01	-0.34499032E+01
2	-0.96478117E+00	-0.27404928E+01
3	-0.75368863E+00	-0.23183076E+01
4	-0.60124397E+00	-0.20134182E+01
5	-0.48081008E+00	-0.17725508E+01
6	-0.38051108E+00	-0.15719523E+01
7	-0.29400149E+00	-0.13989335E+01
8	-0.21748431E+00	-0.12458992E+01
9	-0.14850001E+00	-0.11079304E+01
10	-0.85358232E-01	-0.98164696E+00
11	-0.26842147E-01	-0.86461544E+00
12	0.27957678E-01	-0.75501478E+00
13	0.79747498E-01	-0.65143549E+00
14	0.12908906E+00	-0.55275208E+00
15	0.17644542E+00	-0.45803934E+00
16	0.22220883E+00	-0.36651289E+00
17	0.26672184E+00	-0.27748659E+00
18	0.31029534E+00	-0.19033924E+00
19	0.35322428E+00	-0.10448690E+00
20	0.39578688E+00	-0.19356817E-01
21	0.43828440E+00	0.65638542E-01
22	0.48103148E+00	0.15113252E+00
23	0.52438730E+00	0.23784402E+00
24	0.56878209E+00	0.32663429E+00
25	0.61476314E+00	0.41859576E+00
26	0.66306633E+00	0.51520199E+00
27	0.71475738E+00	0.61858422E+00
28	0.77151549E+00	0.73209941E+00
29	0.83630300E+00	0.86167556E+00
30	0.91535592E+00	0.10197815E+01
31	0.10269278E+01	0.12429250E+01
32	0.17178835E+01	0.26248367E+01

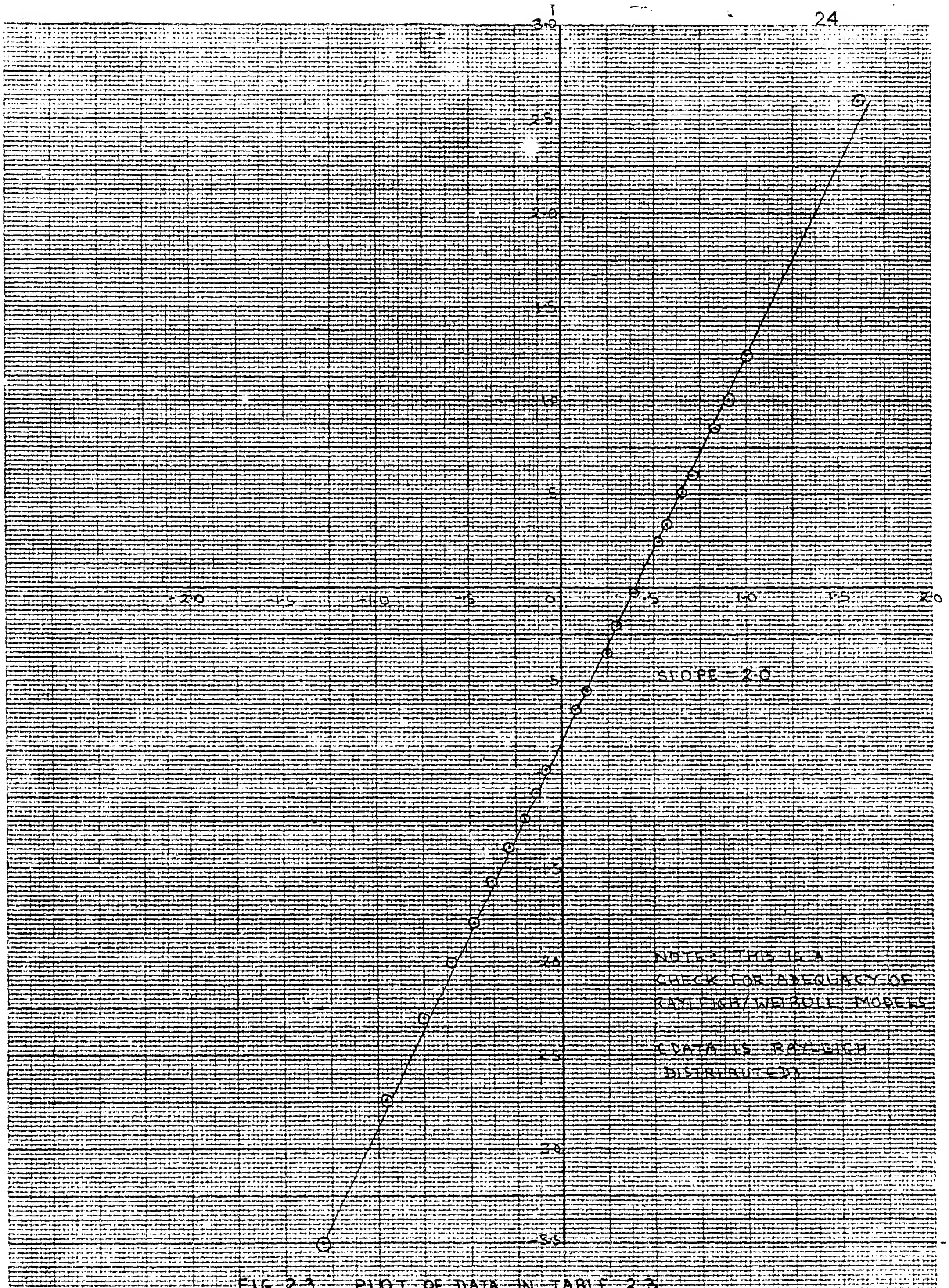


FIG 2.3 PLOT OF DATA IN TABLE 2.2

TABLE 2.4
LOGNORMAL DISTRIBUTION TEST : OUTPUT DATA

	X	Y
1	0.71114302E-02	0.31250000E-01
2	0.30459225E-01	0.62500000E-01
3	0.62957972E-01	0.93750000E-01
4	0.99884063E-01	0.12500000E+00
5	0.13870990E+00	0.15625000E+00
6	0.17804623E+00	0.18750000E+00
7	0.21711010E+00	0.21875000E+00
8	0.25545776E+00	0.25000000E+00
9	0.29284254E+00	0.28125000E+00
10	0.32913667E+00	0.31250000E+00
11	0.36428505E+00	0.34375000E+00
12	0.39827728E+00	0.37500000E+00
13	0.43113327E+00	0.40625000E+00
14	0.46288955E+00	0.43750000E+00
15	0.49359494E+00	0.46875000E+00
16	0.52330554E+00	0.50000000E+00
17	0.55208200E+00	0.53125000E+00
18	0.57998896E+00	0.56250000E+00
19	0.60709590E+00	0.59375000E+00
20	0.63346857E+00	0.62500000E+00
21	0.65918660E+00	0.65625000E+00
22	0.68432975E+00	0.68750000E+00
23	0.70898724E+00	0.71875000E+00
24	0.73326164E+00	0.75000000E+00
25	0.75727671E+00	0.78125000E+00
26	0.78118807E+00	0.81250000E+00
27	0.80520743E+00	0.84375000E+00
28	0.82964867E+00	0.87500000E+00
29	0.85503119E+00	0.90625000E+00
30	0.88237929E+00	0.93750000E+00
31	0.91444021E+00	0.96875000E+00
32	0.99367672E+00	0.99999899E+00

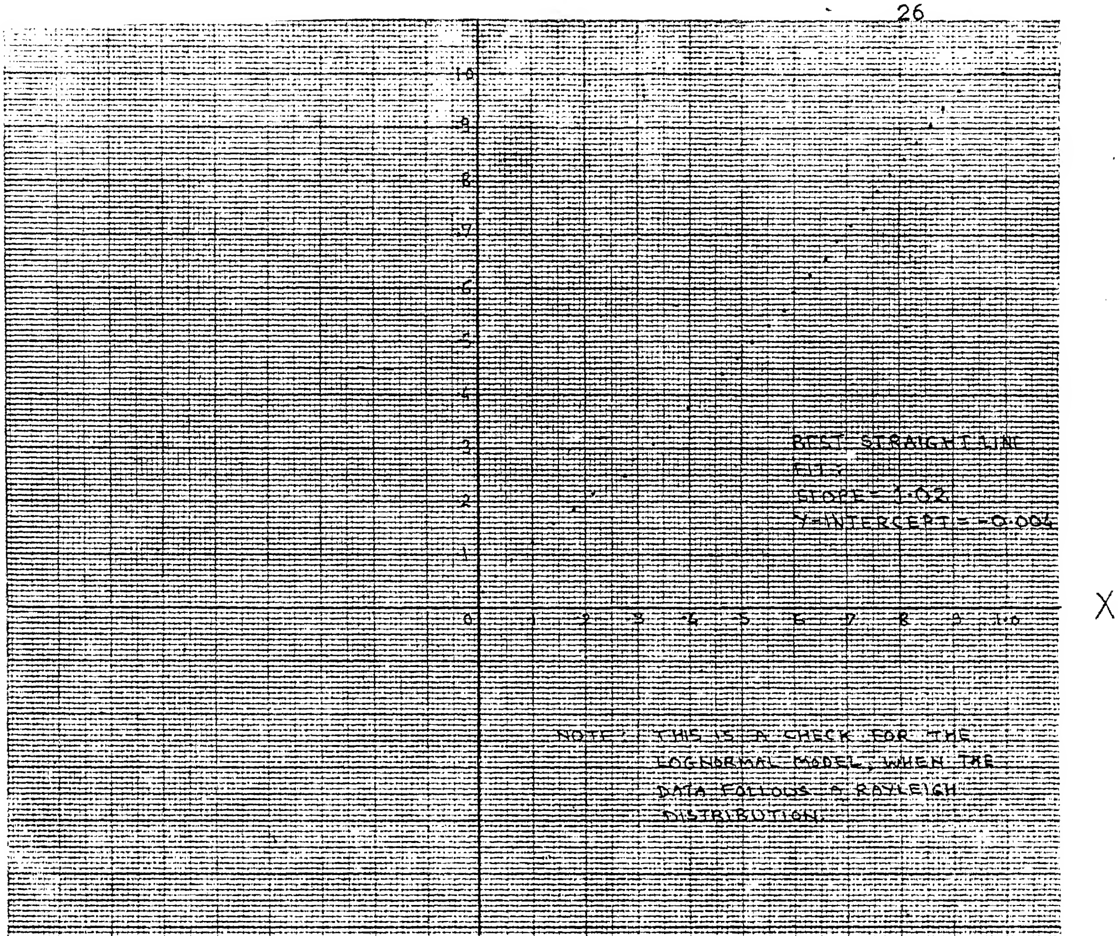


FIG 2.4 PLOT OF DATA IN TABLE 2.4

the estimation of the mean and variance of the ln of the data magnitudes from the data itself. Therefore, a large number of sample values (e.g. N = 1024) should be taken in order to obtain a correct assessment (when subjecting the cell data to the lognormal test program segment). This is further illustrated in the following example.

Example 2:

Here we consider a sequence of N = 1024 samples of the complex reflectivity $\tilde{b}_H$, of a particular range cell H. The magnitude values of the complex samples follow a lognormal distribution (within an absolute error of $\pm$ 1.0 x 10^{-6} due to the data generation method). The distributional parameters are:

$$\text{mean of } \ln |\tilde{b}_H| = \mu = 0.0$$

$$\text{variance of } \ln |\tilde{b}_H| = \sigma^2 = 1.0$$

Table 2.5 shows 32 of these 1024 sample values beginning with the 32nd sample followed by samples at equispaced intervals of 32 samples.

No time-averaging was done on the data.

The data was first assumed to satisfy a Weibull distribution. The 1024 point data was processed by the Rayleigh/Weibull test program segment. The resultant data is shown* in Table 2.6 and its plot in Fig. 2.5. We conclude from the plot that the Weibull model does not fit the data.

Next the data was assumed to satisfy a lognormal distribution. The 1024 point data was processed by the lognormal test program segment. The resultant data is shown* in Table 2.7 and its plot in Fig. 2.6. The plot is a straight line with

$$\text{slope} = 1.0107476$$

$$\text{Y-axis intercept} = -0.475845 \times 10^{-2}$$

indicating the model to be an adequate description of the clutter data. The estimates of μ and σ^2 obtained from the data are:

$$\mu_E = 0.00278$$

$$\sigma_E^2 = 1.02$$

These may be compared with the corresponding estimates obtained when using only the N = 32 data values of Table 2.5:

$$\mu_E = 0.1417$$

$$\sigma_E^2 = 1.4145$$

*only 32 output values out of 1024, beginning with the 32nd value, followed at intervals of 32 values.

TABLE 2.5
CELL DATA

	IN-PHASE PART	QUADRATURE PART
1	0.64382300E+01	0.19654922E+00
2	0.92446465E+02	0.11341490E+02
3	0.34001467E+01	0.15480714E+01
4	0.90991777E+00	0.45470920E+01
5	-0.12445091E+01	-0.24473181E+01
6	0.30067203E+01	0.96994948E+00
7	0.94849020E+00	0.19558196E+01
8	-0.22073731E+01	-0.10117555E+01
9	-0.66622227E-01	-0.17832400E+01
10	0.14229839E+01	0.13522596E+01
11	-0.12694244E+01	0.79002595E+00
12	-0.38186795E+00	0.15849701E+01
13	0.12897211E+00	-0.12611200E+01
14	0.36822775E+00	0.13250426E+01
15	-0.40718207E+00	-0.10019920E+01
16	0.11641711E+01	-0.12018557E+00
17	0.92286336E+00	0.56398459E-01
18	0.97726899E+00	0.21200335E+00
19	0.60236287E+00	0.50933009E+00
20	-0.50095046E+00	0.69218123E+00
21	0.60773802E+00	0.27922726E+00
22	0.91490082E-01	0.72135979E+00
23	-0.10879235E+00	-0.54972440E+00
24	0.22325191E+00	0.57130373E+00
25	-0.35265395E+00	-0.29543519E+00
26	-0.90000726E-01	-0.50140363E+00
27	0.33962879E+00	0.13157070E+00
28	0.36092383E+00	-0.19833487E+00
29	-0.26728570E+00	-0.14299947E-01
30	-0.28947607E+00	0.12803377E+00
31	0.14460196E+00	0.56497019E-01
32	0.17777355E+00	0.12206038E+00

TABLE 2.6

WEIBULL DISTRIBUTION TEST : OUTPUT DATA

	X	Y
1	-0.18627377E+01	-0.34499032E+01
2	-0.15341281E+01	-0.27404928E+01
3	-0.13180081E+01	-0.23183076E+01
4	-0.11503482E+01	-0.20134182E+01
5	-0.10099884E+01	-0.17725508E+01
6	-0.88714826E+00	-0.15719523E+01
7	-0.77641833E+00	-0.13989335E+01
8	-0.67448819E+00	-0.12458992E+01
9	-0.57912910E+00	-0.11079304E+01
0	-0.48877808E+00	-0.98164696E+00
1	-0.40224794E+00	-0.86461544E+00
2	-0.31863827E+00	-0.75501478E+00
3	-0.23720007E+00	-0.65143549E+00
4	-0.15731005E+00	-0.55275208E+00
5	-0.78410178E-01	-0.45803934E+00
6	0.00000000E+00	-0.36651289E+00
7	0.78410089E-01	-0.27748659E+00
8	0.15731007E+00	-0.19033924E+00
9	0.23720244E+00	-0.10448690E+00
0	0.31864005E+00	-0.19356817E-01
1	0.40225020E+00	0.65638542E-01
2	0.48877823E+00	0.15113252E+00
3	0.57912934E+00	0.23784402E+00
	0.67448950E+00	0.32663429E+00
	0.77641940E+00	0.41859576E+00
	0.88714939E+00	0.51520199E+00
	0.10099896E+01	0.61858422E+00
	0.11503501E+01	0.73209941E+00
	0.13180094E+01	0.86167556E+00
	0.15341194E+01	0.10197815E+01
	0.18627195E+01	0.12429250E+01
	0.45340991E+01	0.26248367E+01

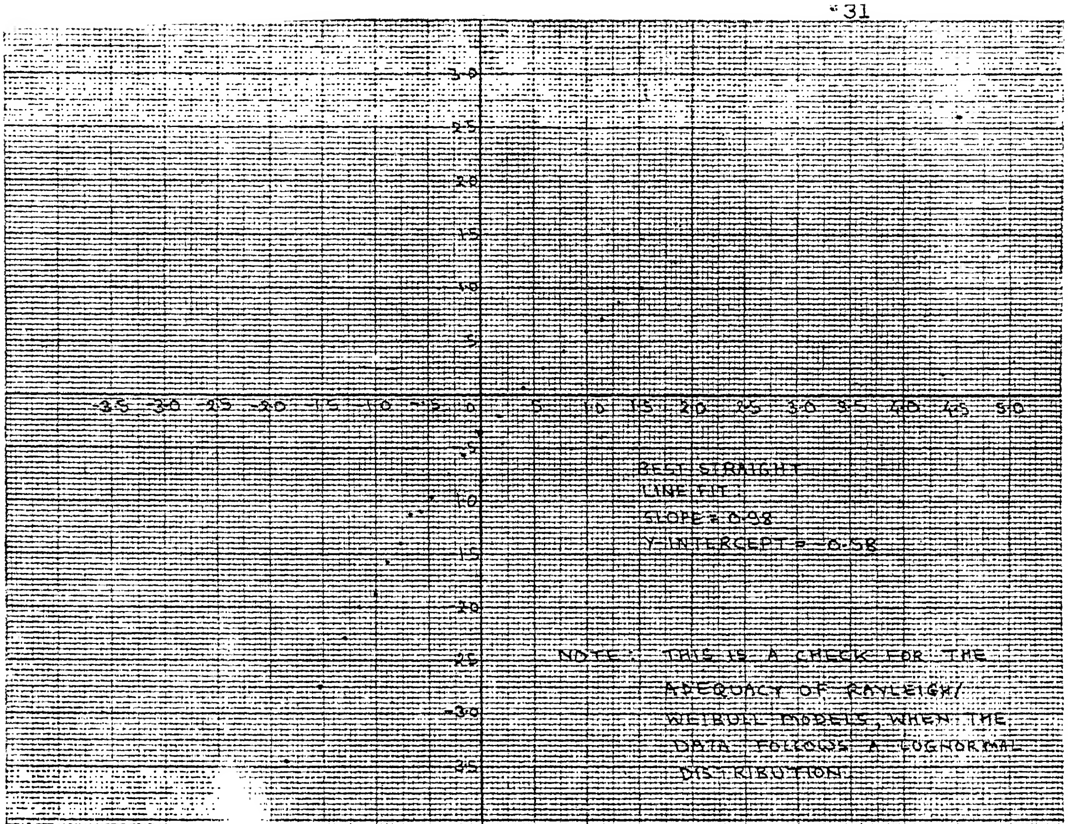


FIG PLOT OF DATA IN TABLE 2.6

TABLE 2.7

LOGNORMAL DISTRIBUTION TEST : OUTPUT DATA

	X	Y
1	0.33704311E-01	0.31250000E-01
2	0.65934449E-01	0.62500000E-01
3	0.97678959E-01	0.93750000E-01
4	0.12912902E+00	0.12500000E+00
5	0.16037679E+00	0.15625000E+00
6	0.19147357E+00	0.18750000E+00
7	0.22245741E+00	0.21875000E+00
8	0.25334883E+00	0.25000000E+00
9	0.28416958E+00	0.28125000E+00
10	0.31493151E+00	0.31250000E+00
11	0.34565195E+00	0.34375000E+00
.2	0.37633678E+00	0.37500000E+00
.3	0.40699780E+00	0.40625000E+00
.4	0.43764171E+00	0.43750000E+00
.5	0.46827841E+00	0.46875000E+00
6	0.49891269E+00	0.50000000E+00
7	0.52955335E+00	0.53125000E+00
8	0.56020939E+00	0.56250000E+00
9	0.59088647E+00	0.59375000E+00
0	0.62159282E+00	0.62500000E+00
1	0.65233701E+00	0.65625000E+00
2	0.68312985E+00	0.68750000E+00
3	0.71397960E+00	0.71875000E+00
4	0.74490404E+00	0.75000000E+00
5	0.77591527E+00	0.78125000E+00
6	0.80703694E+00	0.81250000E+00
7	0.83829159E+00	0.84375000E+00
8	0.86972016E+00	0.87500000E+00
9	0.90137768E+00	0.90625000E+00
0	0.93336284E+00	0.93750000E+00
1	0.96588403E+00	0.96875000E+00
2	0.99999565E+00	0.99999899E+00

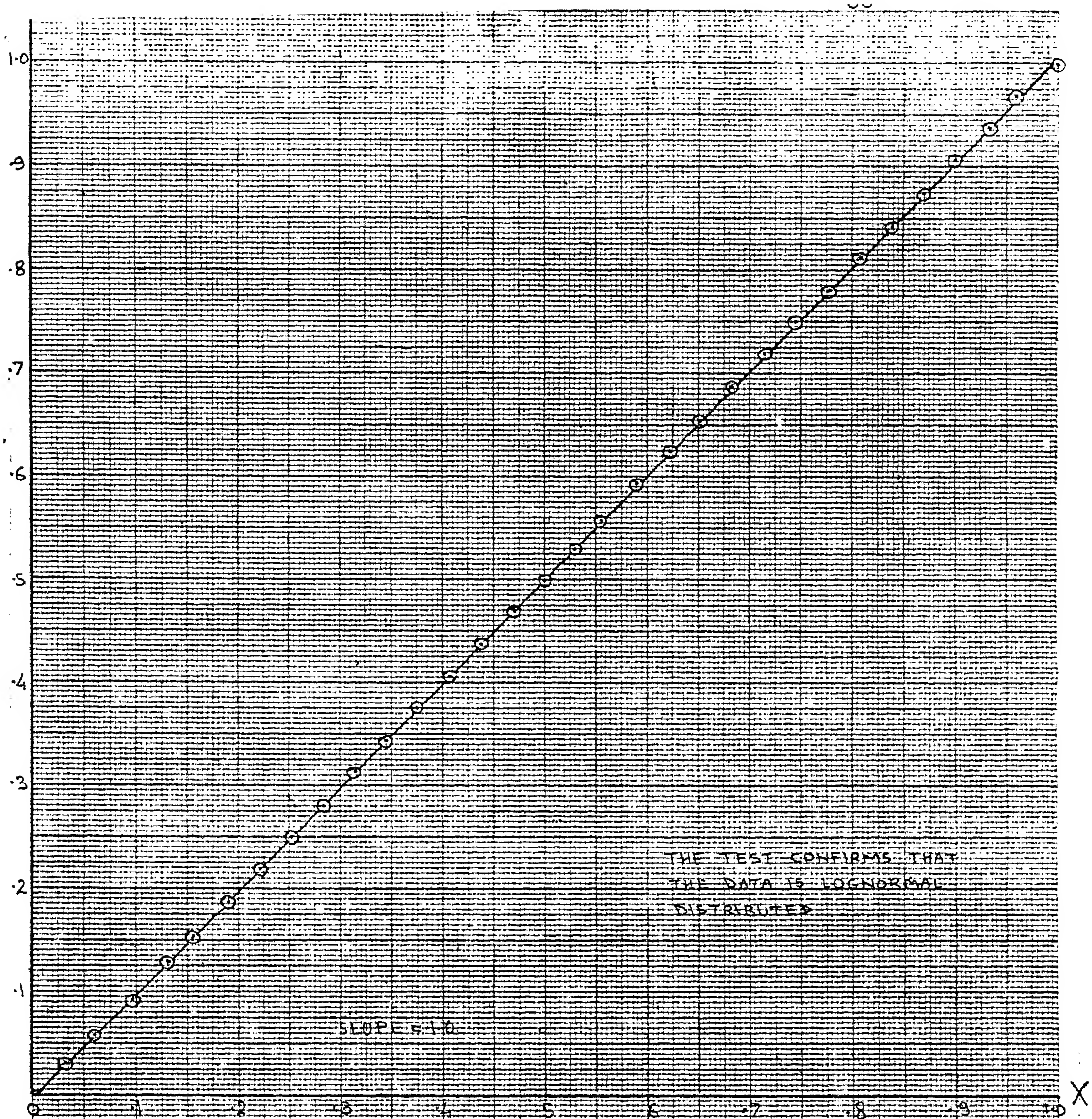


FIG 2.6 PLOT OF DATA IN TABLE 2.7

CHAPTER 3

CLUTTER POWER SPECTRAL DENSITY AND SPATIAL CORRELATION ESTIMATION

3.1 INTRODUCTION:

In this chapter methods for the clutter power spectral density and spatial correlation estimation are presented. Adopting a scattering function model for a doubly spread target, an approximate model using tapped-delay lines is described. A method for finding the PSD of a clutter range cell using DFTs is given. The scattering function model assumes a sufficient spatial separation between the cells for their cross correlation to be negligible. As this need not be the case, two methods of estimating the cross correlation are also given.

3.2 POWER SPECTRAL DENSITY OF THE CLUTTER FROM A RANGE CELL: ESTIMATION OF

3.2.1 Scattering - function characterization of doubly spread targets:

Clutter can be classified according to its spectral characteristics. The spectral width of distributed clutter is related to the nature of fluctuations of its component scatterers. A scattering function whose independent variables are both frequency (doppler shift) and range (or the two way time delay) can be derived starting from (1.1) [2], under the assumptions:

a) $E[\tilde{b}(t,\lambda)] = 0$, where λ is the two way time delay. (3.1)

This assumption does not entail any loss of generality.

b) The $\tilde{b}$ processes from range intervals corresponding to delays λ and λ_1 are statistically uncorrelated

$$E[\tilde{b}(t,\lambda)\,\tilde{b}^*(u,\lambda_1)] = 0, \quad \lambda \neq \lambda_1 \tag{3.2}$$

c) The processes $\tilde{b}(t,\lambda)$ and $\tilde{b}(u,\lambda_1)$ are jointly WSS.

We can define a two-variable covariance function $K(Z,\lambda)$

$$E[\tilde{b}(t+Z,\lambda)\,\tilde{b}^*(t,\lambda_1)] = K(Z,\lambda)\,\delta(\lambda-\lambda_1) \tag{3.3}$$

The covariance function of the clutter signal $\tilde{s}(t)$ in (1.1) is

$$K_{\tilde{s}}(t_1,t_2) = E_t \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}(t_1-\lambda_1)\, E[\tilde{b}(t_1,\lambda_1)\tilde{b}^*(t_2,\lambda_2)]$$

$$\tilde{f}^*(t_2-\lambda_2)\,d\lambda_1\, d\lambda_2 \tag{3.4}$$

$$= E_t \int_{-\infty}^{\infty} \tilde{f}(t_1-\lambda_1) K(t_1-t_2,\lambda_1) \tilde{f}^*(t_2-\lambda_1)\,d\lambda_1 \tag{3.5}$$

Thus, a second-order characterization of the process $\tilde{s}(t)$ is obtained.

The Fourier transform of $K(Z,\lambda)$ is the scattering function,

$$S(f,\lambda) = \int_{-\infty}^{\infty} K(Z,\lambda)\, e^{-j2\pi f z}\, dz \tag{3.6}$$

Some properties of the scattering function are:

1. The average energy received is given by

$$E_r = \int_{-\infty}^{\infty} K_{\tilde{s}}(t,t)dt \tag{3.7}$$

$$= E_t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f,\lambda)|\tilde{f}(t-\lambda)|^2 \, df d\lambda dt \tag{3.8}$$

whence

$$\frac{E_r}{E_t} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f,\lambda) df d\lambda \tag{3.9}$$

since the energy of the transmitted signal is E_t and

$$\int_{-\infty}^{\infty} |\tilde{f}(t)|^2 dt = 1 \tag{3.10}$$

2. The normalised power density function in the $f-\lambda$ plane is given by

$$g_N(f,\lambda) = \frac{S(f,\lambda)}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f,\lambda) df d\lambda} \tag{3.11}$$

Some models have been suggested in the literature for the clutter power spectral density

$$p(f) = \int_{-\infty}^{\infty} g_N(f,\lambda) d\lambda \tag{3.12}$$

For example [7]

$$p(f) = \frac{1}{1+(f/f_c)^{\alpha}} \tag{3.13}$$

where f_c is the upper cutoff frequency

α is a constant

The value $\alpha = 3.0$ was earlier suggested [8]. More recent work [7] indicates that α would vary with the type of clutter, and α of the order of $\alpha = 1.23$ describes land clutter more closely than $\alpha = 3.0$.

3.2.2 The tapped-delay line model for a doubly-spread target:

An approximate model for a doubly-spread target that is relatively easy to implement is a tapped-delay line model described here.

The complex envelope of the composite clutter signal (1.1) is

$$\tilde{s}(t) = \sqrt{E_t} \int_{-\infty}^{\infty} \tilde{f}(t-\lambda)\tilde{b}(t,\lambda)\,d\lambda\,, \quad -\infty<t<\infty \tag{3.14}$$

Assuming the transmitted signal $\tilde{f}(t)$ is bandlimited,

$$\tilde{F}\{f\} = 0, \quad |f| > \frac{W}{2} \tag{3.15}$$

where $\tilde{F}\{f\}$ is the Fourier transform of $\tilde{f}(t)$, we expand $\tilde{f}(t-\lambda)$ using the sampling theorem

$$\tilde{f}(t-\lambda) = \sum_{k=-\infty}^{\infty} \tilde{f}\left(t - \frac{k}{W_s}\right)\left(\frac{\sin\pi W_s(\lambda - k/W_s)}{\pi W_s(\lambda - k/W_s)}\right) \tag{3.16}$$

where $W_s = W$

Substituting (3.16) into (3.14), we have

$$\tilde{s}(t) = \sum_{k=-\infty}^{\infty} \tilde{f}\left(t - \frac{k}{W_s}\right)\tilde{b}_k(t) \tag{3.17}$$

where,

$$\tilde{b}_k(t) \triangleq \int_{-\infty}^{\infty} \tilde{b}(t,\lambda) \frac{\sin\pi W_s(\lambda-k/W_s)}{\pi W_s(\lambda-k/W_s)} d\lambda \tag{3.18}$$

The target model given by (3.17) is shown in Fig. 3.1. The cross-covariance function of the tap gains, using (3.3) and (3.18), is given by

$$E[\tilde{b}_k(t)\, \tilde{b}_\ell^*(u)] = \int_{-\infty}^{\infty} K(t-u,\lambda) \frac{\sin\pi W_s(\lambda-k/W_s)}{\pi W_s(\lambda-k/W_s)} \cdot \frac{\sin\pi W_s(\lambda-\ell/W_s)}{\pi W_s(\lambda-\ell/W_s)} \tag{3.19}$$

Assuming $K(t-u,\lambda)$ to be constant with respect to λ over $1/W_s$,

$$E[\tilde{b}_k(t)\tilde{b}_\ell^*(u)] = \begin{cases} 0, & k \neq \ell \\ \frac{1}{W_s} K(t-u,\, k/W_s)\ , & k = \ell \end{cases} \tag{3.20}$$

The tap gain processes can be characterized in terms of their spectra, by taking the Fourier transform of (3.20),

$$S_k(f) = \frac{1}{W_s} S(f, \frac{k}{W_s}) \tag{3.21}$$

$S_k(f)$ is just the cross-section of the scattering function $S(f,\lambda)$ at various values of λ .

We, thus, have an approximate model for the target,

$$\tilde{s}_R(t) = \sum_{k=0}^{R} \tilde{f}(t - \frac{k}{W_s})\tilde{b}_k(t) \tag{3.22}$$

3.2.3 Estimation of relative power spectral density from the data:

The power spectral estimates of the clutter data measured

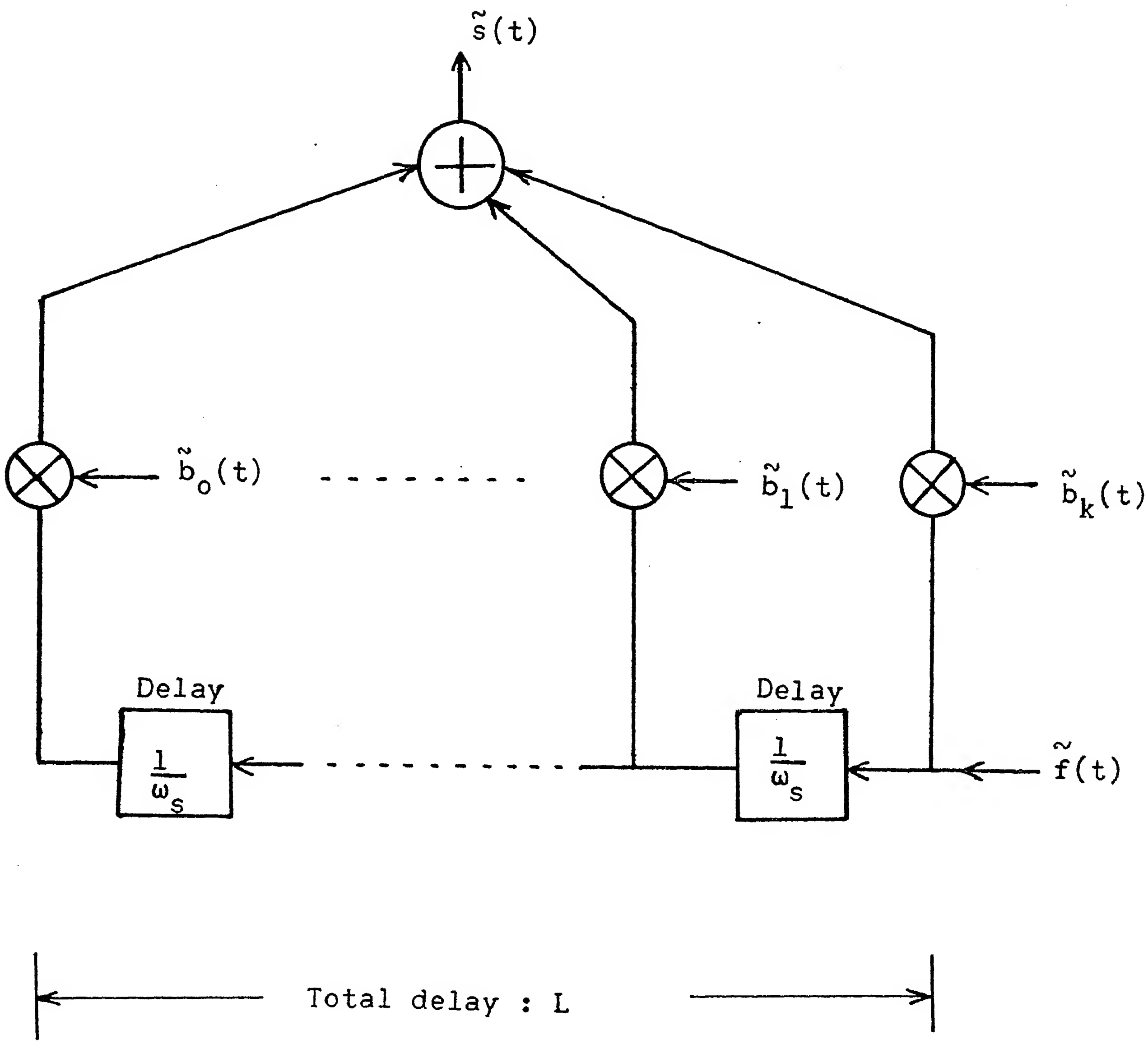


Fig. 3.1: TAPPED-DELAY LINE MODEL FOR A DOUBLY-SPREAD TARGET

from the Kth range cell are obtained by finding the square of the modulus of the DFT of the $\{\tilde{b}_K(n)\}$ sequence, i.e.

$$\tilde{b}_K(n) \Longleftrightarrow \tilde{B}_K(m)$$

$$n = 1,\ldots,N \qquad m = 0,1,\ldots,N-1 \tag{3.23}$$

$$G_K(m) = |\tilde{B}_K(m)|^2, \; m = 0,1,\ldots,N-1 \tag{3.24}$$

where $G_K(m)$ denotes the relative power spectrum.

It is convenient to write (3.24) in the form of a probability density

$$g_K(m) = \frac{G_K(m)}{\sum_{m=o}^{N-1} |\tilde{B}_K(m)|^2} \tag{3.25}$$

The cumulative density function can be similarly found. The various steps in the evaluation of the normalised PSD function are shown as a flow chart in Fig. 3.2.

A radix 2, in-place, decimation-in-time FFT algorithm has been used for the computation of the DFT and is shown in the form of a flow-chart in Fig. 3.3. The algorithm is explained in terms of a decimation-in-time signal flow graph in Fig. 3.4.

The resolution in both frequency and range, with respect to the measurement system in Table 1.1, obtained are:

a) range resolution $R_o = \frac{1}{2}C\,T_o$

where C is the velocity of light

T_o is the range interval in two-way time delay units

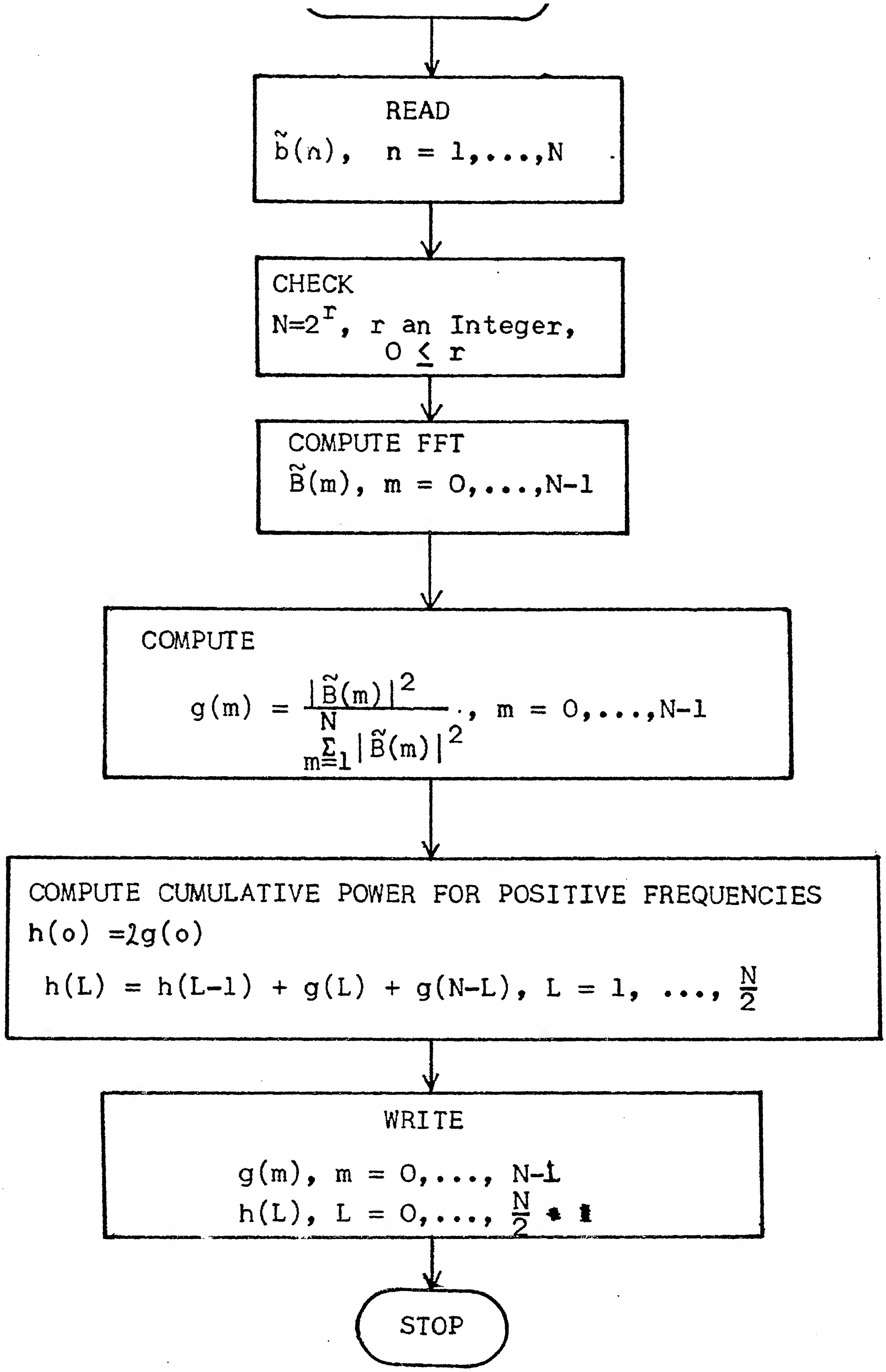


Fig. 3.2: EVALUATION OF THE POWER SPECTRAL DENSITY OF THE CLUTTER FROM ONE RANGE CELL.

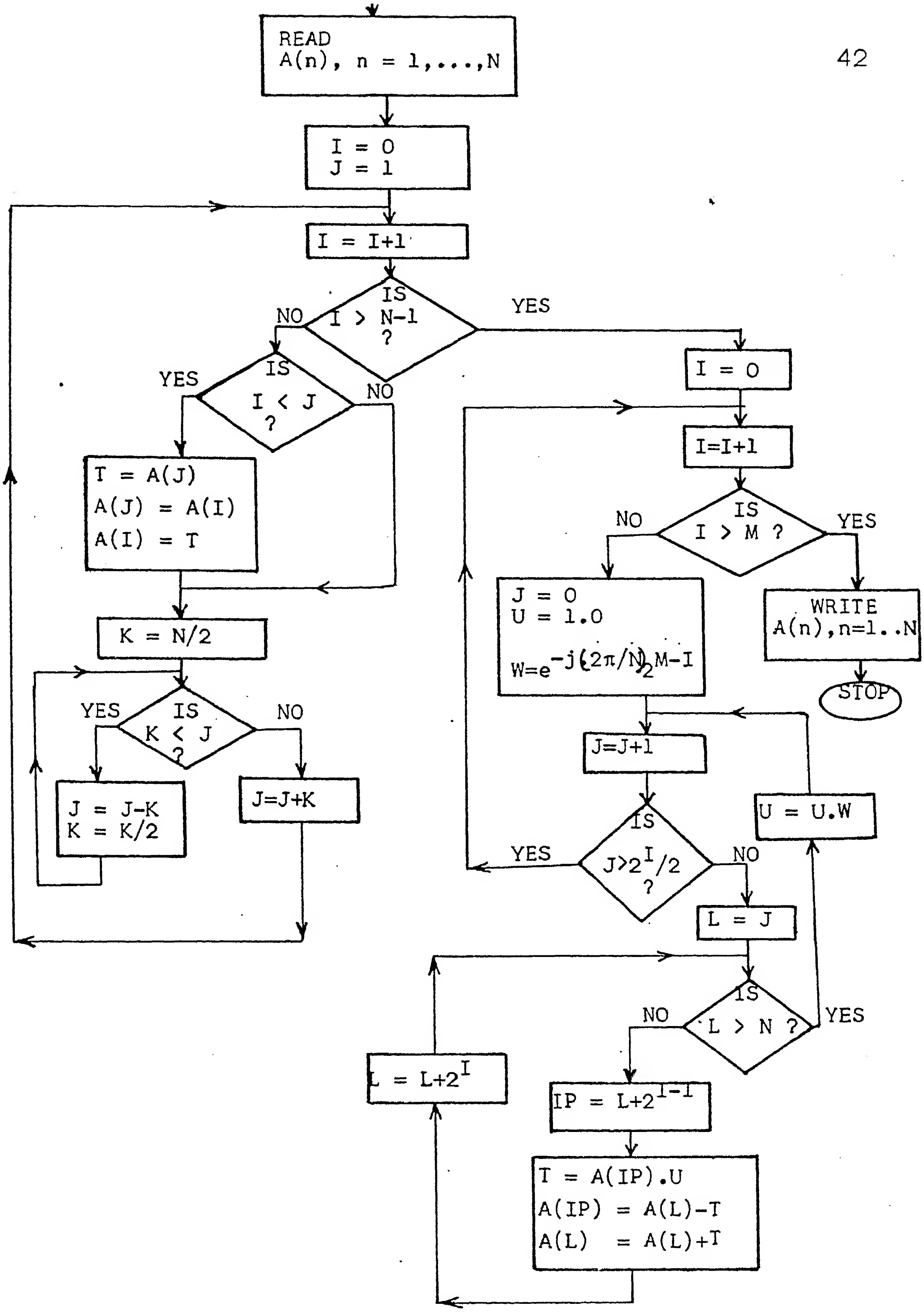
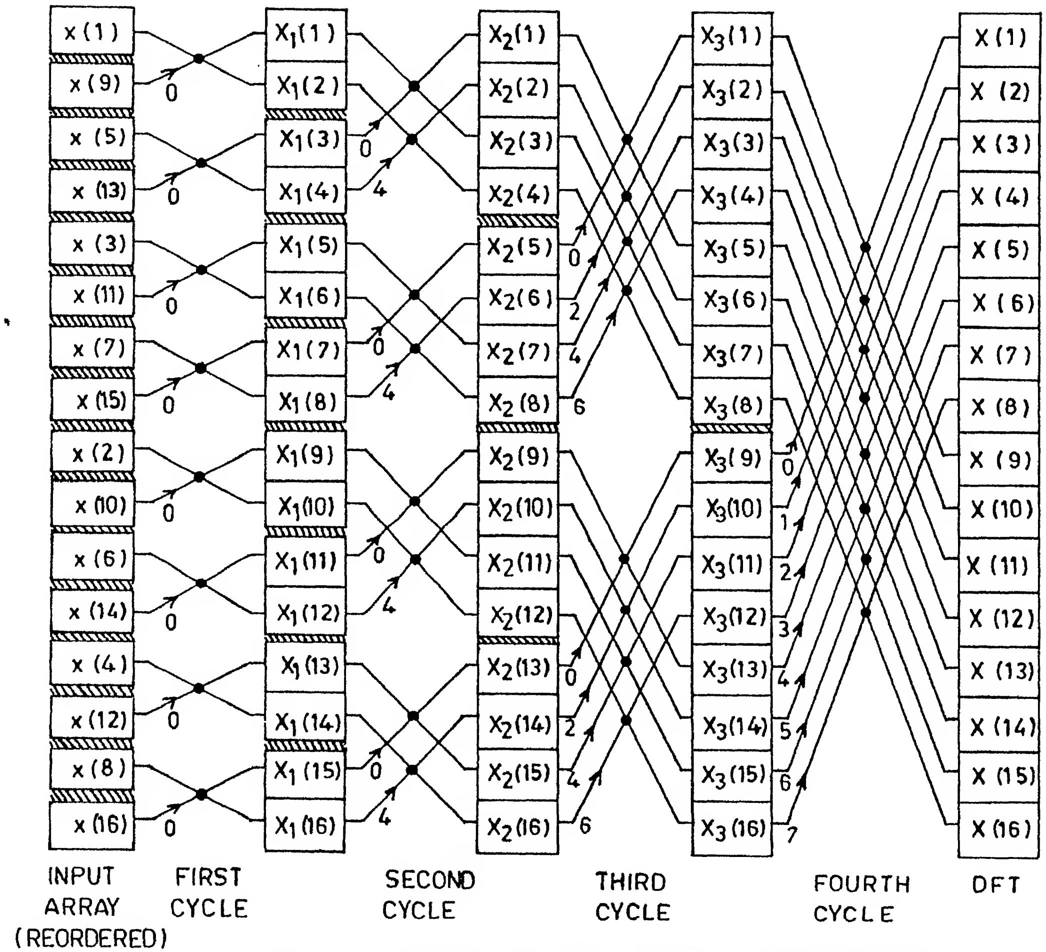


Fig. 3.3: FFT ALGORITHM



$x(n) \Longleftrightarrow X(k)$

$1 \leq n \leq N$ $\quad$ $1 \leq k \leq N$

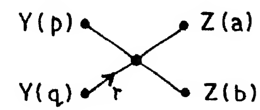
Where

$$X(k) = \sum_{n=1}^{N} x(n) e^{-j \frac{2\pi}{N}(n-1)(k-1)}$$

$1 \leq k \leq N$

NOTE :

1. The starting array locations are used for storing the resultant array in each cycle.
2. The butterfly



denotes

$Y(p) + W_N^r \, Y(q) = Z(a)$

$Y(p) - W_N^r \, Y(q) = Z(b)$

Where

$$W_N = e^{-j \frac{2\pi}{N}}$$

FIG.3.4 SIGNAL FLOW GRAPH (DECIMATION - IN - TIME) FOR THE FFT (N=16)

For $T_o = 0.1\ \mu s$, $R_o = 15$ m

b) frequency resolution $f_o = \frac{1}{NT}$

where T is the interval between samples, T = 3.27670 sec

N is the no. of samples in the sequence, $N = 2^r$, r being an integer.

The highest recoverable frequency $f_H = \frac{1}{2T}$

$= 152.59$ Hz.

For N = 1024, $f_o = 0.298$ Hz

An illustrative example of PSD estimation using a program incorporating the scheme of Fig. 3.2 is shown at the end of the chapter.

3.3 SPATIAL CORRELATION ESTIMATION:

3.3.1 Cross Correlation Function:

The cross correlation between the clutter from any two range cells is given by the function

$$R_{K,L}(t_1,t_2) = E\,[\tilde{b}_K(t_1) \cdot \tilde{b}_L^{\,*}(t_2)] \tag{3.26}$$

where K and L are the range cell indices.

Assuming the processes to be jointly wide-sense stationary, we can write (3.26) as

$$R_{K,L}(u) = E[\tilde{b}_K\,(t + u)\ \tilde{b}_L^{\,*}(t)] \tag{3.27}$$

In general $R_{K,L}(u)$ would depend upon:

1. the relative rate of fluctuation, density and nature of the point scatterers in the two range cells,
2. the spatial separation between the cells,
3. the temporal separation u.

A study of the correlation properties of the various sources of clutter is useful in the estimation of clutter from a particular range cell by using the clutter data obtained from adjoining cell(s). These estimates could be used in the detection of weak radar signals, for the purpose of clutter cancellation in the cell of interest,

3.3.2 Evaluation by the Direct Method:

The normalised cross covariance (called correlation coefficient henceforth, after Papoulis [6]) between the continuous complex random processes $\tilde{b}_K$ and $\tilde{b}_L$ is the quantity to be evaluated and is given by

$$\tilde{\rho}_{K,L}(u) = \frac{E\{[\tilde{b}_K(t+u)-\tilde{\mu}_K][\tilde{b}_L(t)-\tilde{\mu}_L]^*\}}{\{E[|\tilde{b}_K(t+u)-\tilde{\mu}_K|^2].E[|\tilde{b}_L(t)-\tilde{\mu}_L|^2]\}^{1/2}} \tag{3.28}$$

where,

$$\tilde{\mu}_K(t+u) = E[\tilde{b}_K(t+u)]$$

$$\tilde{\mu}_L(t) = E[\tilde{b}_L(t)]$$

The data consists of the two sequences

$$\begin{aligned} &\text{Kth range cell} : \tilde{b}_K(n),\ n = 1,\ldots,N \\ &\text{Lth range cell} : \tilde{b}_L(n),\ n = 1,\ldots,N \end{aligned} \tag{3.29}$$

The value of $\tilde{\rho}$ is estimated as

$$\tilde{\rho}_{K,L}(M) = \frac{\sum_{n=o}^{I-1} [\{\tilde{b}_K(n+N_1)-\tilde{\mu}_K\}\{\tilde{b}_L(n+N_1+M)-\tilde{\mu}_L\}^*]}{\left\{[\sum_{n=o}^{I-1} |\tilde{b}_K(n+N_1)-\tilde{\mu}_K|^2][\sum_{n=o}^{I-1} |\tilde{b}_L(n+N_1+M)-\tilde{\mu}_L|^2]\right\}^{1/2}} \tag{3.30}$$

where,

$$\tilde{\mu}_K = \frac{1}{I} \sum_{n=o}^{I-1} \tilde{b}_K(n+N_1)$$

$$\tilde{\mu}_L = \frac{1}{I} \sum_{n=o}^{I-1} \tilde{b}_L(n+N_1+M)$$

N_1,M and L are variable parameters having integer values:

N_1 : index of the first sample to be taken in the first sequence, for the computation of $\tilde{\rho}$, $1 \leq N_1 \leq N$

M : no. of samples corresponding to the time difference $u = t_1-t_2$, $0 \leq M \leq N-N_1$.

I : no. of samples corresponding to the time frame over which $\tilde{\rho}$ is to be evaluated, $2 \leq I \leq N-N_1-M+1$

Varying N_1, in effect, enables a shift along the time-axis, of the interval over which $\tilde{\rho}$ is estimated, while varying I varies the length of this interval. Varying M, in effect, changes the time difference ($u = t_1-t_2$) in increments or decrements equal to integer multiples of the sample-to-sample interval.

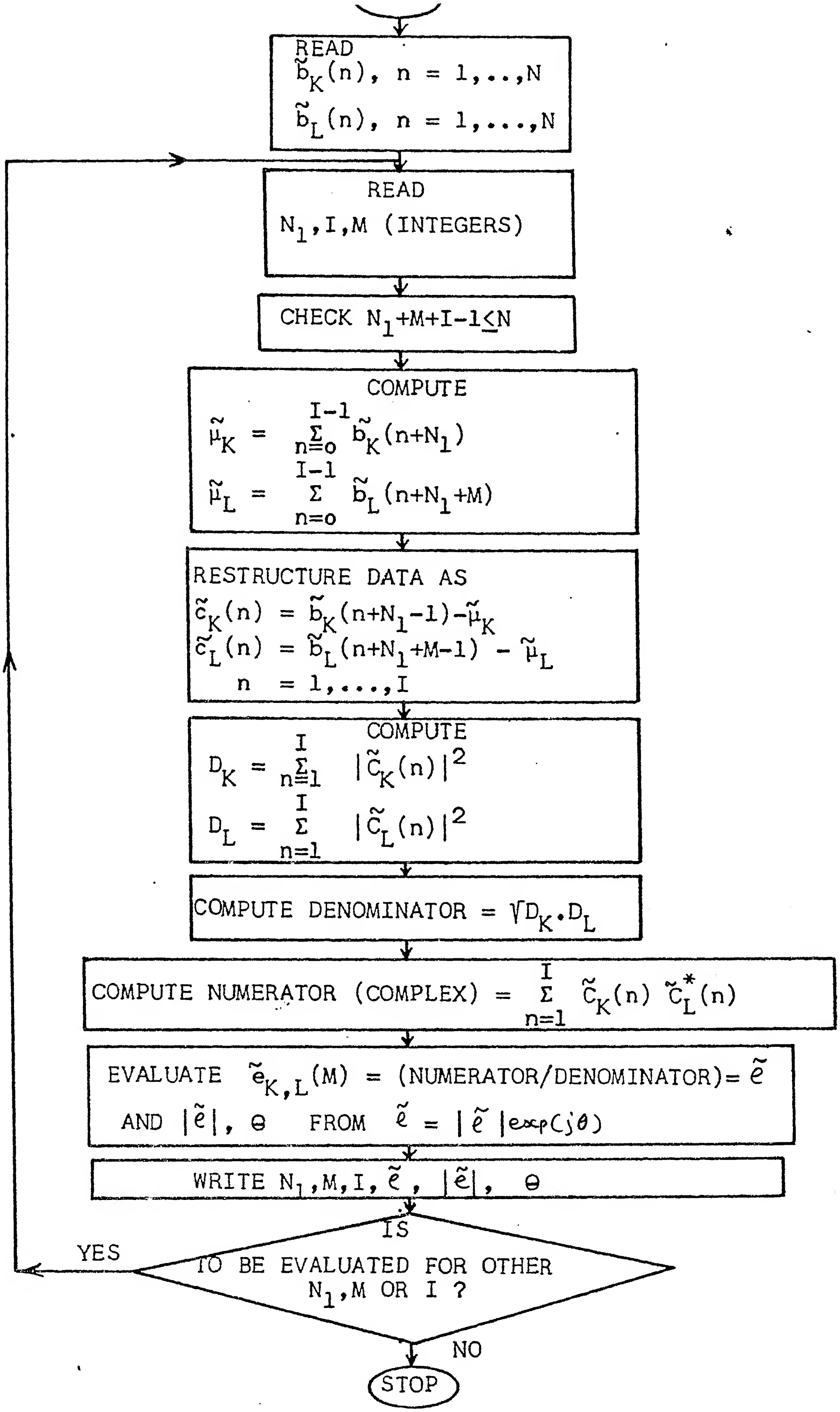


Fig. 3.5: EVALUATION OF CROSS COVARIANCE (NORMALISED) BETWEEN THE COMPLEX REFLECTIVITY PROCESS OF TWO RANGE CELLS: DIRECT METHOD

The various steps in the evaluation of $\tilde{s}_{K,L}(M) = \tilde{\rho}$ are shown in the flow chart of Fig. 3.5.

3.3.3 Evaluation by the FFT Method:

The direct (conventional) method described in the previous section is suitable when the no. of data points used in the computation (I) is small, i.e. when $I \leq 64$, as a thumb-rule[10]. Again, only one value of $\tilde{\rho}_{K,L}(M)$ is computed at a time, corresponding to a preselected value of N_1, M and I. In case a series of values

$$\tilde{\rho}_{K,L}(M), \; M = M_1, \ldots, M_j$$

is required, the method is cumbersome as N_1, M and I have to be fed in each time.

When it is required to evaluate the correlation function over all possible values of M and for large values of I ($I > 64$ as a thumb-rule), a faster method employing Fast Fourier Transforms can be used.

For convenience we change the symbols for the two sequences as follows, and we specify $N = 2^J$, where J can take non-negative integer values,

$$\left.\begin{aligned} \tilde{b}_K(n) &= x(n) \\ \tilde{b}_L(n) &= y(n) \end{aligned}\right\} \begin{array}{l} n = 1,\ldots,N;\ x \text{ and } y \text{ denoting} \\ \text{complex numbers} \end{array} \qquad (3.31)$$

Since sample indices are numbered n=1,...,N, rather than from n=0, the Discrete Fourier Transform (DFT) can be written as

$$F[x(n)] = \sum_{n=1}^{N} x(n)\, e^{-j2\pi(n-1)(k-1)/N}, \quad 1 \le k \le N \qquad (3.32)$$
$$1 \le n \le N$$
$$\triangleq X(k), \quad 1 \le k \le N$$

The correlation function corresponding to the changed notation (3.31) is

$$z(\ell) = \frac{1}{N} \sum_{n=1}^{N} x(n+\ell)\, y^*(n), \quad 0 \le \ell \le N-1 \qquad (3.33)$$

The DFT pair corresponding to discrete-time correlation is

$$\frac{1}{N} \sum_{n=1}^{N} x(n+\ell)\, y^*(n) \Longleftrightarrow \frac{1}{N} X(k)\, Y^*(k) \qquad (3.34)$$

$$0 \le l \le N-1, \qquad 1 \le k \le N$$
$$1 \le n \le N$$

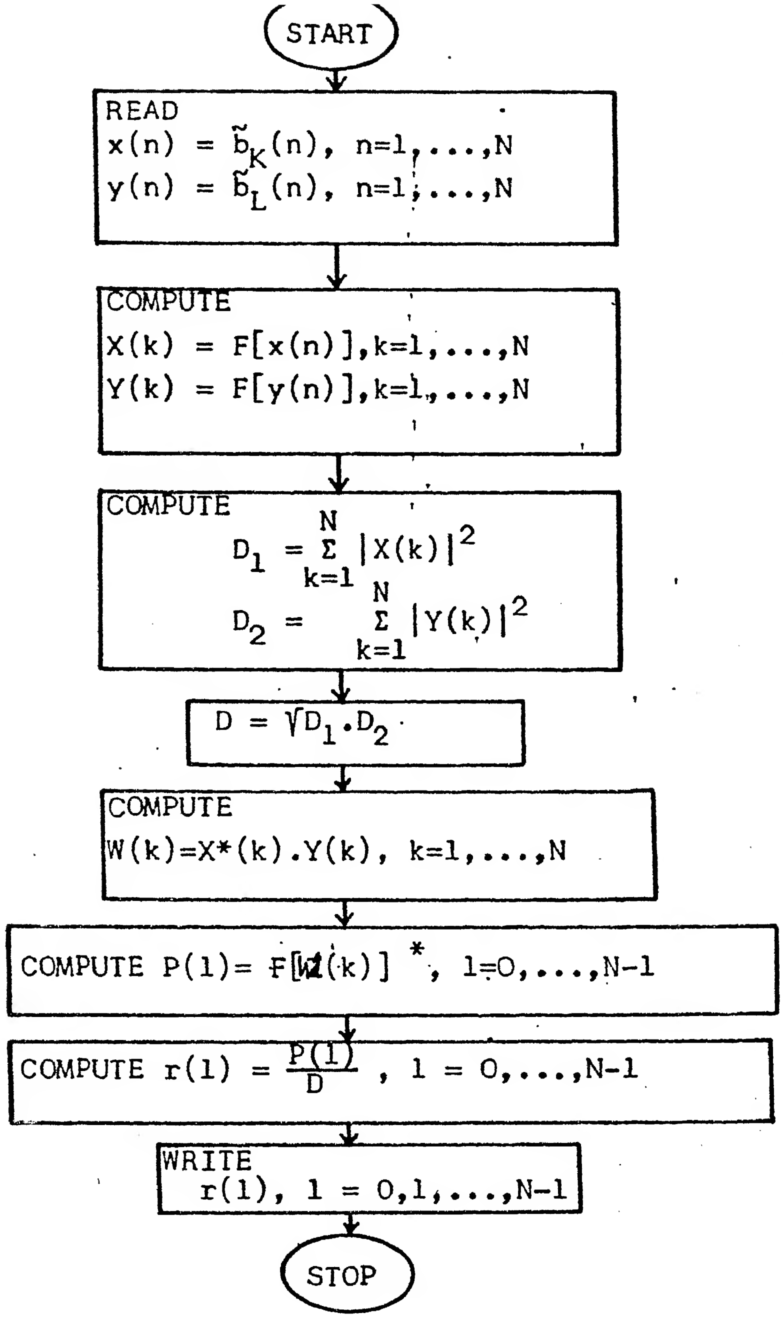
In order to use (3.34) the two data sequences are assumed cyclic with period N, i.e.

$$\left.\begin{aligned} x(n+N) &= x(n) \\ y(n+N) &= y(n) \end{aligned}\right\} n = 1,\ldots,N \qquad (3.35)$$

The normalised value of the cross-correlation

$$r(\ell) = \frac{z(\ell)}{\frac{1}{N}\left\{[\sum_{n=1}^{N} |x(n)|^2][\sum_{n=1}^{N} |y(n)|^2\right\}^{1/2}}, \quad 0 \le \ell \le N-1$$
$$(3.36)$$

is evaluated here (as compared to the normalised cross_covariance in the first method) through the steps shown in Fig. 3.6. The specific FFT algorithm used is the same as for PSD estimation.



NOTE: 1. The DFT and the inverse DFT are both computed by the same 'fast' algorithm by means of the relation

$$F^{-1}[X(k)] = \frac{1}{N}\Big[\sum_{k=1}^{N} X^*(k)e^{-j2\pi(k-1)(n-1)/N}\Big]^*, \quad n=1,\ldots,N$$

$$= \frac{1}{N}\, F[X^*(k)]^*, \quad n=1,\ldots,N$$

2. $x(n) = b_K(n)$, $y(n) = b_L(n)$ $n=1,\ldots,N$

LEGEND: F[.] denotes the DFT of the sequence within brackets.

Fig. 3.6: Evaluation of cross-correlation (Normalized) between the complex reflectivity processess of two range cells: FFT method.

3.4 SOME ILLUSTRATIVE EXAMPLES USING GENERATED DATA

3.4.1 Estimation of correlation from the data:

The following cases are considered:

1. Autocorrelation of the data from one range cell (by the FFT method).
2. Cross correlation between the data from two range cells

   a) when positively perfectly correlated

   b) when negatively perfectly correlated

   (by both methods).
3. Cross correlation between the data from two range cells; when the value of correlation is low (by the FFT method).

Example 1:

The autocorrelation properties of the data obtained from a particular range can be investigated. We consider the data of example 2, Chapter II (Table 2.5), where the magnitude values follow a lognormal distribution. The auto-correlation of this complex-valued sequence, evaluated by the FFT method, is shown in Table 3.1. As expected, normalised correlation values are symmetrical about the value of time shift index

$$M = \frac{N}{2} = 16$$

Since the methods assumes the sequence to be periodic.

TABLE 3.1

C O R R E L A T I O N (F F T M E T H O D)

r is the correlation value (normalised).

x	Re(r)	Im(r)	Abs(r)	Phase(r) (deg)
	0.99999994E+00	0.00000000E+00	0.99999994E+00	0.00000000E+00
	0.10448679E+00	0.22927677E-01	0.10697275E+00	0.12376337E+02
	0.17978633E-01	0.43906569E-01	0.47444895E-01	0.67732132E+02
	-0.10911529E-01	-0.23324009E-01	0.25750162E-01	0.64928680E+02
	0.30328518E-01	0.52912477E-02	0.30786626E-01	0.98964691E+01
	0.90166871E-02	0.20706305E-01	0.22584327E-01	0.66468987E+02
	-0.19834511E-01	-0.59564873E-02	0.20709602E-01	0.16715500E+02
	0.26362482E-03	-0.19726947E-01	0.19728709E-01	-0.89234360E+02
	0.15620577E-01	0.18456591E-01	0.24179500E-01	0.49757378E+02
	-0.16632585E-01	0.13054860E-01	0.21144083E-01	-0.38128246E+02
	0.11454485E-02	0.11694972E-01	0.11750934E-01	0.84406082E+02
	-0.20606765E-02	-0.68387045E-02	0.71424269E-02	0.73231102E+02
	0.83971350E-02	0.45178696E-02	0.95353564E-02	0.28281368E+02
	0.80722850E-03	-0.12252662E-01	0.12279224E-01	-0.86230690E+02
	0.93083074E-02	-0.12592740E-01	0.15659556E-01	-0.53528912E+02
	0.19427773E-01	-0.54777153E-02	0.20185236E-01	-0.15745947E+02
	0.22854388E-01	0.00000000E+00	0.22854388E-01	0.00000000E+00
	0.19427821E-01	0.54777395E-02	0.20185290E-01	0.15745977E+02
	0.93083056E-02	0.12592763E-01	0.15659573E-01	0.53528969E+02
	0.80722431E-03	0.12252644E-01	0.12279205E-01	0.86230705E+02
	0.83971424E-02	-0.45178626E-02	0.95353592E-02	-0.28281311E+02
	-0.20606755E-02	0.68387082E-02	0.71424306E-02	-0.73231117E+02
	0.11454439E-02	-0.11694972E-01	0.11750932E-01	-0.84406097E+02
	-0.16632600E-01	-0.13054851E-01	0.21144090E-01	0.38128204E+02
	0.15620572E-01	-0.18456591E-01	0.24179496E-01	-0.49757389E+02
	0.26363973E-03	0.19726932E-01	0.19728694E-01	0.89234314E+02
	-0.19834507E-01	0.59564891E-02	0.20709598E-01	-0.16715508E+02
	0.90166861E-02	-0.20706305E-01	0.22584327E-01	-0.66468987E+02
	0.30328525E-01	-0.52912547E-02	0.30786633E-01	-0.98964787E+01
	-0.10911531E-01	0.23324024E-01	0.25750175E-01	-0.64928688E+02
	0.17978638E-01	-0.43906596E-01	0.47444925E-01	-0.67732140E+02
	0.10448688E+00	-0.22927696E-01	0.10697283E+00	-0.12376337E+02

te: Re(.), Im(.), Abs(.) denote the real part, imaginary part and absolute value respectively.

Example 2:

We consider a hypothetical case where the clutter variations as measured from two range cells K and L are similar in all respects, except, due to the relative locations of K and L with respect to the radar, for a scaling factor S = 3.0. The data from cell K is taken as the data of example 1, Chapter II (Table 2.2), where the magnitude values follow a Rayleigh distribution. The number of samples (N)=32. The data from cell L is shown in Table 3.2. Results of correlation performed by the direct method, for a few values of the parameters N_1, M and L, are shown in Table 3.3, where,

N_1 = starting sample number in the first sequence;

$1 \leq N_1 \leq N$

M = time delay; $0 \leq M \leq N - 1$

L = number of consecutive samples over which correlation is evaluated; $1 \leq L \leq N$,

subject to the condition $N_1 + M + L - 1 \leq N$, which is checked by the program. Results of correlation performed by the FFT method are shown in Table 3.4. For $M > 0$, there is a difference as expected in the values given by the two methods, since normalised cross covariance and normalised cross correlation are respectively arrived at by the direct and FFT program segments, and since dc content is present in the input sequence. (In the FFT program segment, $N_1=1$ and L=N are fixed values, and correlation is evaluated for all M, $0 \leq M \leq N-1$).

TABLE 3.2
CELL DATA

	IN PHASE PART	QUADRATURE PART
1	0.83735199E+01	0.25563064E+00
2	0.16593754E+02	0.20357499E+01
3	0.63011022E+01	0.28688633E+01
4	0.14702725E+01	0.73473282E+01
5	-0.27790837E+01	-0.54650474E+01
6	0.61757326E+01	0.19922533E+01
7	0.24207370E+01	0.49916439E+01
8	-0.52927208E+01	-0.24259326E+01
9	-0.18921965E+00	-0.50647368E+01
10	0.38407242E+01	0.36498349E+01
11	-0.39480035E+01	0.24570386E+01
12	-0.11367635E+01	0.47182178E+01
13	0.43451491E+00	-0.42487903E+01
14	0.11932796E+01	0.42939358E+01
15	-0.14746627E+01	-0.36288435E+01
16	0.40698533E+01	-0.42015955E+00
17	0.35722437E+01	0.21830861E+00
18	0.36613345E+01	0.79426968E+00
19	0.24810073E+01	0.20978248E+01
20	-0.20012326E+01	0.27651749E+01
21	0.26538374E+01	0.12193145E+01
22	0.38816822E+00	0.30605390E+01
23	-0.50204134E+00	-0.25367994E+01
24	0.10025810E+01	0.25656142E+01
25	-0.17138822E+01	-0.14358016E+01
26	-0.42642385E+00	-0.23756530E+01
27	0.17295971E+01	0.67003828E+00
28	0.17970763E+01	-0.98752940E+00
29	-0.14098659E+01	-0.75428672E-01
30	-0.15038581E+01	0.66514868E+00
31	0.74683809E+00	0.29179499E+00
32	0.94243771E+00	0.64708340E+00

TABLE 3.3

Cross-covariance (Normalised) (r) by Direct Method (Example 2)

(a) $M = 0$, $N_1 = 1$

L	Re(r)	Im(r)	Abs(r)	Ph(r)(deg)
8	0.10000000E+01	-0.38345824E-08	0.10000000E+01	-0.21970537E-06
16	0.10000000E+01	0.60278986E-08	0.10000000E+01	0.34537314E-06
24	0.10000002E+01	-0.15261712E-10	0.10000002E+01	-0.87443142E-09
32	0.10000000E+01	0.65132508E-08	0.10000000E+01	0.37318176E-06

(b) $N_1 = 1$, $L = 16$

M	Re(r)	Im(r)	Abs(r)	Ph(r) (deg)
4	0.27008435E+00	-0.17715546E+00	0.32300094E+00	-0.33261925E+02
8	0.21851349E+00	-0.28608727E+00	0.35999176E+00	-0.52627403E+02
12	0.56510989E-01	-0.10714775E-01	0.57517804E-01	-0.10736131E+02
16	0.22118418E+00	-0.58230016E-01	0.22872075E+00	-0.14749292E+02

(c) $M = 0$, $L = 16$

N_1	Re(r)	Im(r)	Abs(r)	Ph(r) (deg)
2	0.99999994E+00	0.22430551E-08	0.99999994E+00	0.12851760E-06
4	0.10000002E+01	-0.18534123E-08	0.10000002E+01	-0.10619267E-06
8	0.99999994E+00	0.42214330E-08	0.99999994E+00	0.24187031E-06
16	0.10000001E+01	-0.63362982E-08	0.10000001E+01	-0.36304306E-06

Note: Re(.), Im(.), Abs(.) and Ph(.) denote the real part, imaginary part, absolute value and phase respectively.

TABLE 3.4

C O R R E L A T I O N (F F T M E T H O D)

r is the correlation value (normalised).

x	Re(r)	Im(r)	Abs(r)	Phase(r) (deg)
	0.10000000E+01	-0.63633991E-08	0.10000000E+01	-0.36459591E-06
	0.21218443E+00	0.31674460E+00	0.38124713E+00	0.56182217E+02
	0.30918017E-01	0.38881302E-01	0.49675744E-01	0.51508636E+02
	0.96674092E-01	-0.12555692E+00	0.15846267E+00	-0.52405083E+02
	0.15747385E+00	0.91264315E-01	0.18200876E+00	0.30094528E+02
	-0.64674772E-01	0.16692807E+00	0.17901900E+00	-0.68821587E+02
	-0.55691682E-01	-0.94160937E-01	0.10939765E+00	0.59397720E+02
	0.31489424E-01	-0.13609625E+00	0.13969171E+00	-0.76972366E+02
	0.10393536E+00	0.21296953E+00	0.23697802E+00	0.63986267E+02
	-0.17572403E+00	0.52376147E-01	0.18336357E+00	-0.16597168E+02
	0.64104125E-01	0.34263592E-01	0.72686538E-01	0.28124462E+02
	0.76061860E-02	0.20459294E-02	0.78765405E-02	0.15055211E+02
	0.11210383E+00	-0.28170757E-01	0.11558920E+00	-0.14105874E+02
	-0.42503420E-01	-0.10358461E+00	0.11196567E+00	0.67690392E+02
	0.17828840E+00	-0.19614510E+00	0.26506537E+00	-0.47730358E+02
	0.28360718E+00	-0.33399165E-01	0.28556705E+00	-0.67165351E+01
	0.20140755E+00	0.61459562E-08	0.20140755E+00	0.17483820E-05
	0.28360704E+00	0.33399343E-01	0.28556693E+00	0.67165737E+01
	0.17828833E+00	0.19614497E+00	0.26506522E+00	0.47730354E+02
	-0.42503323E-01	0.10358448E+00	0.11196552E+00	-0.67690414E+02
	0.11210384E+00	0.28170794E-01	0.11558921E+00	0.14105891E+02
	0.76061189E-02	-0.20459145E-02	0.78764716E-02	-0.15055234E+02
	0.64104088E-01	-0.34263633E-01	0.72686531E-01	-0.28124502E+02
	-0.17572403E+00	-0.52376114E-01	0.18336356E+00	0.16597157E+02
	0.10393530E+00	-0.21296950E+00	0.23697796E+00	-0.63986275E+02
	0.31489499E-01	0.13609622E+00	0.13969171E+00	0.76972336E+02
	-0.55691671E-01	0.94160996E-01	0.10939769E+00	-0.59397739E+02
	-0.64674839E-01	-0.16692810E+00	0.17901906E+00	0.68821571E+02
	0.15747385E+00	-0.91264337E-01	0.18200877E+00	-0.30094536E+02
	0.96674219E-01	0.12555696E+00	0.15846279E+00	0.52405056E+02
	0.30917995E-01	-0.38881190E-01	0.49675643E-01	-0.51508575E+02
	0.21218443E+00	-0.31674477E+00	0.38124728E+00	-0.56182232E+02

Example 3:

Here we consider two range cells O and P whose clutter returns are 180^{o} out of phase but otherwise similar. The data from Cell O is shown in Table 3.5. The magnitude values of this cell data are Weibull distributed with parameters $\eta = 1.0$ and $\sigma = 3.0$. The data from cell P, shown in Table 3.6, is a scaled by a factor of $S = -0.2$ version of the same data. Results of correlation evaluated by the two methods are shown at

(a) table 3.7 for the direct method (for various N_1, M and L values)

(b) table 3.8 for the FFT method (for all values of M, with $N_1 = 1$ and $L = N$).

Example 4:

While examples 2 and 3 were hypothetical cases meant primarily to check the program functioning, here we consider a case more likely to represent a real situation. We consider two range cells Q and R. The data consists of $N = 32$ complex samples. The magnitude values follow a Weibull distribution, but with different parameter values for the two cells:

Cell Q : $\eta = 1.0$ $\sigma = 3.0$ (Table 3.5, example 3)

Cell R : $\eta = 4.0$ $\sigma = 7.0$ (Table 3.9)

The FFT method is used to obtain estimates of the normalised cross correlation, and the values obtained are given in Table 3.10.

TABLE 3.5
CELL DATA

	IN-PHASE PART	QUADRATURE PART
1	0.10392370E+02	0.31726301E+00
2	0.41098835E+02	0.50420752E+01
3	0.64630241E+01	0.29425857E+01
4	0.16321096E+01	0.81560698E+01
5	-0.25242603E+01	-0.49639392E+01
6	0.59370475E+01	0.19152551E+01
7	0.19895430E+01	0.41025066E+01
8	-0.45652270E+01	-0.20924840E+01
9	-0.14207643E+00	-0.38028808E+01
10	0.30147388E+01	0.28649023E+01
11	-0.27198162E+01	0.16926768E+01
12	-0.81732905E+00	0.33923824E+01
13	0.27493137E+00	-0.26883445E+01
14	0.78785747E+00	0.28350518E+01
15	-0.85574841E+00	-0.21058221E+01
16	0.24669244E+01	-0.25467795E+00
17	0.18940344E+01	0.11574912E+00
18	0.20321739E+01	0.44084862E+00
19	0.11942064E+01	0.10097655E+01
20	-0.10119936E+01	0.13983078E+01
21	0.11482440E+01	0.52756453E+00
22	0.17741053E+00	0.13988055E+01
23	-0.19233757E+00	-0.97187585E+00
24	0.40913469E+00	0.10469795E+01
25	-0.56769508E+00	-0.47558549E+00
26	-0.15247783E+00	-0.84947026E+00
27	0.47527939E+00	0.18412115E+00
28	0.54592150E+00	-0.29999480E+00
29	-0.29489833E+00	-0.15777240E-01
30	-0.36635947E+00	0.16203891E+00
31	0.88715233E-01	0.34661673E-01
32	0.15961395E+00	0.10959189E+00

TABLE 3.6
CELL DATA

	IN-PHASE PART	QUADRATURE PART
1	-0.20784740E+01	-0.63452601E-01
2	-0.82197676E+01	-0.10084151E+01
3	-0.12926048E+01	-0.58851713E+00
4	-0.32642195E+00	-0.16312140E+01
5	0.50485206E+00	0.99278784E+00
6	-0.11874095E+01	-0.38305101E+00
7	-0.39790860E+00	-0.82050133E+00
8	0.91304541E+00	0.41849682E+00
9	0.28415287E-01	0.76057619E+00
10	-0.60294777E+00	-0.57298046E+00
11	0.54396325E+00	-0.33853537E+00
12	0.16346581E+00	-0.67847651E+00
13	-0.54986276E-01	0.53766888E+00
14	-0.15757149E+00	-0.56701034E+00
15	0.17114969E+00	0.42116442E+00
16	-0.49338490E+00	0.50935593E-01
17	-0.37880689E+00	-0.23149824E-01
18	-0.40643477E+00	-0.88169724E-01
19	-0.23884128E+00	-0.20195310E+00
20	0.20239873E+00	-0.27966157E+00
21	-0.22964881E+00	-0.10551291E+00
22	-0.35482105E-01	-0.27976111E+00
23	0.38467515E-01	0.19437517E+00
24	-0.81826940E-01	-0.20939592E+00
25	0.11353902E+00	0.95117100E-01
26	0.30495567E-01	0.16989405E+00
27	-0.95055878E-01	-0.36824230E-01
28	-0.10918430E+00	0.59998959E-01
29	0.58979668E-01	0.31554480E-02
30	0.73271893E-01	-0.32407783E-01
31	-0.17743047E-01	-0.69323345E-02
32	-0.31922791E-01	-0.21918377E-01

TABLE 3.7

ross-covariance (Normalised) (r) by Direct Method (Example 3)

M	L	Re(r)	Im(r)	Abs(r)	Ph(r) (deg)
0	32	-0.10000001E+01	-0.10531747E-07	0.10000001E+01	0.60342455E-06
1	31	-0.26604116E+00	0.14804119E+00	0.30445704E+00	-0.29094175E+02
1	30	-0.24358593E+00	0.30391911E+00	0.38948807E+00	-0.51288425E+02
2	29	-0.57376287E-03	0.30231792E+00	0.30231848E+00	-0.89891251E+02
2	28	0.32842219E+00	-0.32444155E+00	0.46165296E+00	-0.44650658E+02
3	27	-0.38023984E+00	-0.31417581E+00	0.49324310E+00	0.39565456E+02
3	26	-0.26693031E+00	-0.35366309E+00	0.44309074E+00	0.52955997E+02
4	25	-0.14688352E+00	0.33424926E+00	0.36509910E+00	-0.66277245E+02

Note: Re(.), Im(.), Abs(.) and Ph(.) denote the real part, imaginary part, absolute value and phase respectively.

TABLE 3.8

C O R R E L A T I O N (F F T M E T H O D)

r is the correlation value (normalised).

x	Re(r)	Im(r)	Abs(r)	Phase(r) (deg)
	-0.10000001E+01	-0.97221760E-08	0.10000001E+01	0.55703958E-06
	-0.30751368E+00	-0.13710648E+00	0.33669403E+00	0.24029934E+02
	-0.48843816E-01	-0.12228604E+00	0.13167989E+00	0.68227112E+02
	0.20435425E-02	0.85913867E-01	0.85938171E-01	0.88637421E+02
	-0.12307041E+00	-0.34315757E-01	0.12776501E+00	0.15580079E+02
	-0.22129023E-01	-0.97114682E-01	0.99603996E-01	0.77163475E+02
	0.68081960E-01	0.36217906E-01	0.77116087E-01	0.28011864E+02
	0.58417339E-02	0.77528045E-01	0.77747822E-01	0.85690910E+02
	-0.69151975E-01	-0.79678833E-01	0.10550220E+00	0.49045815E+02
	0.65738618E-01	-0.53357881E-01	0.84667757E-01	-0.39065105E+02
	-0.61195716E-02	-0.48783273E-01	0.49165606E-01	0.82849930E+02
	0.63128937E-02	0.19574270E-01	0.20567078E-01	0.72124977E+02
	-0.41378930E-01	-0.37245434E-02	0.41546214E-01	0.51433682E+01
	0.18722257E-02	0.46327148E-01	0.46364959E-01	0.87685753E+02
	-0.52954089E-01	0.66271946E-01	0.84829867E-01	-0.51373665E+02
	-0.99242724E-01	0.24040259E-01	0.10211293E+00	-0.13616860E+02
	-0.10419881E+00	-0.13445560E-07	0.10419881E+00	0.73933070E-05
	-0.99242792E-01	-0.24040390E-01	0.10211304E+00	0.13616923E+02
	-0.52954048E-01	-0.66271976E-01	0.84829867E-01	0.51373695E+02
	0.18722089E-02	-0.46327055E-01	0.46364866E-01	-0.87685776E+02
	-0.41378967E-01	0.37245322E-02	0.41546252E-01	-0.51433487E+01
	0.63128658E-02	-0.19574266E-01	0.20567065E-01	-0.72125053E+02
	-0.61195884E-02	0.48783258E-01	0.49165592E-01	-0.82849907E+02
	0.65738663E-01	0.53357873E-01	0.84667794E-01	0.39065079E+02
	-0.69151931E-01	0.79678833E-01	0.10550217E+00	-0.49045834E+02
	0.58416724E-02	-0.77527978E-01	0.77747747E-01	-0.85690948E+02
	0.68081953E-01	-0.36217898E-01	0.77116080E-01	-0.280118[illegible]+02
	-0.22129010E-01	0.97114697E-01	0.99604003E-01	-0.77163[illegible]02
	-0.12307042E+00	0.34315806E-01	0.12776501E+00	-0.15580[illegible]+02
	0.20435220E-02	-0.85913897E-01	0.85938200E-01	-0.88637436E+02
	-0.48843820E-01	0.12228608E+00	0.13167992E+00	-0.68227119E+02
	-0.30751389E+00	0.13710658E+00	0.33669424E+00	-0.24029938E+02

TABLE 3.10

C O R R E L A T I O N (F F T M E T H O D)

r is the correlation value (normalised).

ex	Re(r)	Im(r)	Abs(r)	Phase(r) (deg)
	-0.32323885E+00	-0.11175871E-06	0.32323885E+00	0.19809815E-04
	-0.11239727E+00	-0.77054560E-01	0.13627380E+00	0.34432800E+02
	-0.25697461E+00	0.13894741E+00	0.29213411E+00	-0.28400305E+02
	-0.32803908E+00	0.11193673E+00	0.34661141E+00	-0.18841173E+02
	0.10873923E+00	0.40536571E-01	0.11604927E+00	0.20444801E+02
	0.20546307E+00	-0.49415704E-01	0.21132199E+00	-0.13523302E+02
	-0.16173336E+00	0.13485540E-01	0.16229460E+00	-0.47663713E+01
	-0.25239691E+00	0.92034005E-02	0.25256464E+00	-0.20883081E+01
	0.59156001E-01	-0.29147747E+00	0.29741982E+00	-0.78527512E+02
	0.23159978E+00	-0.44877812E-01	0.23590778E+00	-0.10966474E+02
	-0.40732965E-01	0.12412541E+00	0.13063802E+00	-0.71832245E+02
	-0.32184020E-01	-0.15193662E+00	0.15530790E+00	0.78040092E+02
	-0.10011426E+00	0.84999911E-01	0.13133106E+00	-0.40332222E+02
	-0.30579131E-01	0.10488811E+00	0.10925475E+00	-0.73746460E+02
	-0.85467711E-01	0.25320908E+00	0.26724443E+00	-0.71348503E+02
	-0.22811495E+00	0.55078831E-01	0.23467019E+00	-0.13574373E+02
	-0.17608058E+00	0.35307392E-01	0.17958559E+00	-0.11338488E+02
	-0.28238043E+00	-0.27339198E-02	0.28239366E+00	0.55470258E+00
	-0.18290828E+00	-0.86243965E-01	0.20222132E+00	0.25244[illegible]02
	0.51903605E-01	-0.11271880E+00	0.12409476E+00	-0.65275[illegible]02
	-0.78484848E-01	0.50584171E-01	0.93373604E-01	-0.32802[illegible]2E+02
	0.19362621E-01	-0.98626427E-01	0.10050911E+00	-0.7889[illegible]792E+02
	-0.64399838E-01	0.95342107E-01	0.11505415E+00	-0.55962524E+02
	0.15022567E+00	0.67049429E-01	0.16450951E+00	0.24052387E+02
	-0.12759882E+00	0.12549472E+00	0.17897034E+00	-0.4452367[illegible]+02
	0.14561258E-01	-0.17772664E+00	0.17832215E+00	-0.853161[illegible]02
	0.11202595E+00	-0.59166439E-01	0.12669049E+00	-0.2784[illegible]+02
	-0.94826967E-02	0.13976103E+00	0.14008234E+00	-0.8611[illegible]E+02
	-0.13102983E+00	0.55986207E-01	0.14248955E+00	-0.2313[illegible]9E+02
	-0.27346574E-02	-0.10144290E+00	0.10147975E+00	0.88455818E+02
	-0.10546800E-01	0.50333917E-01	0.51427018E-01	-0.78165649E+02
	-0.69952682E-01	0.13261861E+00	0.14993690E+00	-0.62189579E+02

As stated earlier the sequences are assumed to be periodic (with periodicity N) and correlation is performed over N samples. The results show a low value of correlation between the two sequences.

3.4.2 Estimation of PSD from the data:

In the following example, the complex reflectivity data consists of a relatively slowly varying sequence, whose PSD is similar to a low-pass-filter transfer characteristic.

Example:

The data sequence considered (Table 3.11) consists of $N = 32$ samples of the complex reflectivity of a given range cell, in which successive sample magnitudes decay exponentially over the interval $0 \leq \eta \leq N/2$. The data is symmetric around $\eta = \frac{N}{2} = 16$. Normalised values of the PSD obtained with the program are shown at Table 3.12. The PSD values are plotted in Fig. 3.7 for the frequency index $0 \leq k \leq 16$, where $\frac{N}{2} = 16$ represents the maximum recoverable frequency. The cell data are also plotted in Fig. 3.7.

Normalised PSD values were also obtained, for purpose of comparison, through direct computation of the DFT (i.e. by 'brute force' method) and these are shown in Table 3.13. As expected, it is seen that the two sets of values of normalised power spectral density in Tables 3.12 and 3.13 are equivalent.

TABLE 3.11

CELL DATA :

x(n) = Re(x(n)) + j*Im(x(n))

Abs(x(n)) is its magnitude

The complex sequence [x(n)] represents a slowly varying clutter signal :

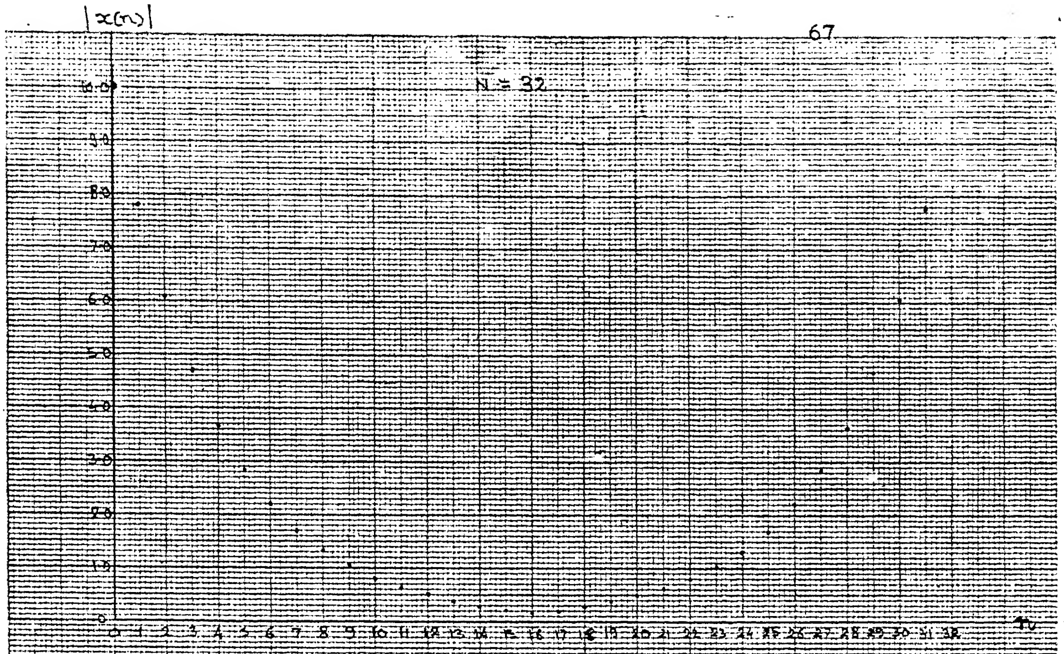
n	Re(x(n))	Im(x(n))	Abs(x(n))
0	0.99999943E+01	0.99999998E-02	0.99999990E+01
1	0.77879934E+01	0.15576007E-01	0.77880092E+01
2	0.60652795E+01	0.18195892E-01	0.60653062E+01
3	0.47236280E+01	0.18894613E-01	0.47236657E+01
4	0.36787486E+01	0.18393897E-01	0.36787946E+01
5	0.28649957E+01	0.17190183E-01	0.28650472E+01
6	0.22312472E+01	0.15618986E-01	0.22313018E+01
7	0.17376839E+01	0.13901768E-01	0.17377394E+01
8	0.13532982E+01	0.12180012E-01	0.13533530E+01
9	0.10539396E+01	0.10539747E-01	0.10539923E+01
	0.82080042E+00	0.90291686E-02	0.82085001E+00
	0.63923275E+00	0.76711616E-02	0.63927877E+00
	0.49782866E+00	0.64721378E-02	0.49787074E+00
	0.38770416E+00	0.54282136E-02	0.38774216E+00
	0.30193993E+00	0.45294384E-02	0.30197391E+00
	0.23514734E+00	0.37626789E-02	0.23517744E+00
	0.18312991E+00	0.31135085E-02	0.18315637E+00
	0.23514734E+00	0.37626789E-02	0.23517744E+00
	0.30193993E+00	0.45294384E-02	0.30197391E+00
	0.38770416E+00	0.54282136E-02	0.38774216E+00
	0.49782866E+00	0.64721378E-02	0.49787074E+00
	0.63923275E+00	0.76711616E-02	0.63927877E+00
	0.82080042E+00	0.90291686E-02	0.82085001E+00
	0.10539396E+01	0.10539747E-01	0.10539923E+01
	0.13532982E+01	0.12180012E-01	0.13533530E+01
	0.17376839E+01	0.13901768E-01	0.17377394E+01
	0.22312472E+01	0.15618986E-01	0.22313018E+01
	0.28649957E+01	0.17190183E-01	0.28650472E+01
	0.36787486E+01	0.18393897E-01	0.36787946E+01
	0.47236280E+01	0.18894613E-01	0.47236657E+01
	0.60652795E+01	0.18195892E-01	0.60653062E+01
	0.77879934E+01	0.15576007E-01	0.77880092E+01

TABLE 3.12

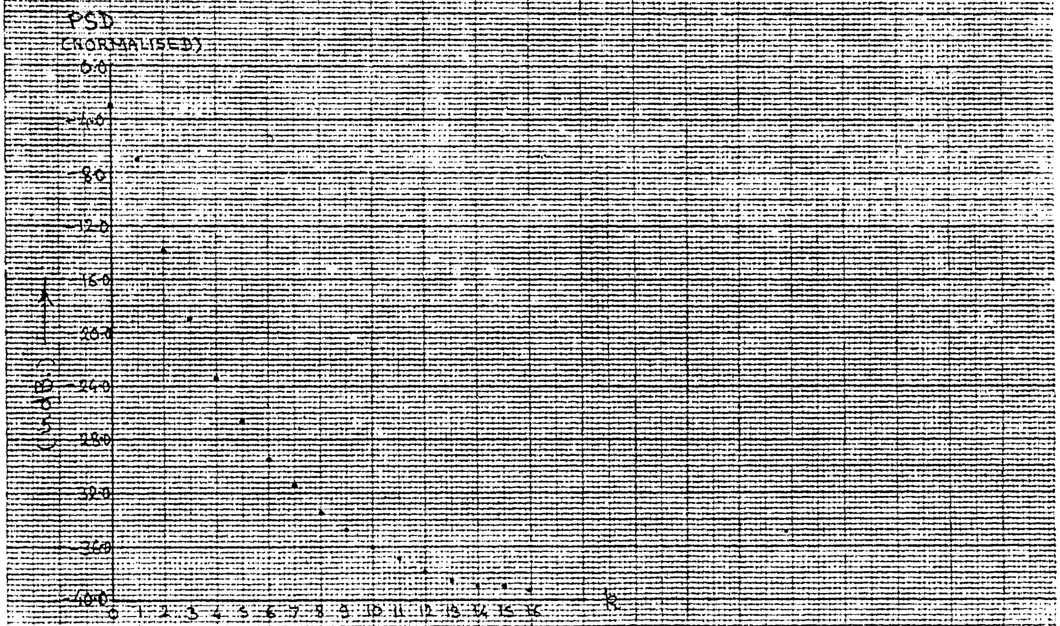
POWER SPECTRAL DENSITY OF THE CLUTTER FROM ONE RANGE CELL

(THE PSD VALUES ARE EVALUATED AS FRACTIONS OF THE TOTAL POWER RETURNED FROM THAT RANGE CELL)

FREQUENCY INDEX (m)	POWER (NORMALISED) (p)	p in dB	CUMULATIVE POWER (dB) (ONE-SIDED)
0	0.47713768E+00	-0.32135630E+01	-0.18978081E+01
1	0.19765860E+00	-0.70408435E+01	-0.39251748E+00
2	0.40718611E-01	-0.13902071E+02	-0.13805725E+00
3	0.12673114E-01	-0.18971165E+02	-0.61808705E-01
4	0.44768206E-02	-0.23490303E+02	-0.35190552E-01
5	0.22375344E-02	-0.26502304E+02	-0.21947678E-01
6	0.11187978E-02	-0.29512487E+02	-0.15340543E-01
7	0.72434766E-03	-0.31400530E+02	-0.11068582E-01
8	0.44260218E-03	-0.33539864E+02	-0.84603112E-02
9	0.33678967E-03	-0.34726414E+02	-0.64767990E-02
10	0.23546191E-03	-0.36280792E+02	-0.50906050E-02
11	0.20116336E-03	-0.36964516E+02	-0.39068400E-02
12	0.15578169E-03	-0.38074833E+02	-0.29899587E-02
13	0.14598809E-03	-0.38356827E+02	-0.21309331E-0[illegible]
14	0.12315245E-03	-0.39095570E+02	-0.14066435E-[illegible]
15	0.12512119E-03	-0.39026695E+02	-0.67096431E-[illegible]
16	0.11409923E-03	-0.39427177E+02	0.00000000E+[illegible]
17	0.12512716E-03	-0.39026485E+02	
18	0.12315485E-03	-0.39095490E+02	
19	0.14598899E-03	-0.38356804E+02	
20	0.15578221E-03	-0.38074821E+02	
21	0.20116351E-03	-0.36964508E+02	
22	0.23546156E-03	-0.36280800E+02	
23	0.33678749E-03	-0.34726440E+02	
24	0.44260218E-03	-0.33539864E+02	
25	0.72434888E-03	-0.31400524E+02	
26	0.11187982E-02	-0.29512484E+02	
27	0.22375348E-02	-0.26502304E+02	
28	0.44768234E-02	-0.23490301E+02	
29	0.12673126E-01	-0.18971165E+02	
30	0.40718656E-01	-0.13902066E+02	
31	0.19765893E+00	-0.70408363E+01	



(A) COMPLEX REFLECTIVITY SAMPLES FROM ONE RANGE CELL (TABLE 3.11)



(B) POWER SPECTRA DENSITY (TABLE 3.12)

FIG 3.7 EVALUATION OF PSD FROM CLUTTER DATA

TABLE 3.13
POWER SPECTRAL DENSITY

(Evaluated by direct computation of the DFT, i.e. by 'Brute Force' method. The program used is given at the end.)

Index k	PSD (normalised)
0	0.47713766E+00
1	0.19765873E+00
2	0.40718615E-01
3	0.12673118E-01
4	0.44768201E-02
5	0.22375358E-02
6	0.11187992E-02
7	0.72434935E-03
8	0.44260250E-03
9	0.33679037E-03
10	0.23546259E-03
11	0.20116477E-03
12	0.15578243E-03
13	0.14598986E-03
14	0.12315555E-03
15	0.12512758E-03
16	0.11409910E-03
17	0.12512758E-03
18	0.12315555E-03
19	0.14598986E-03
20	0.15578243E-03
21	0.20116489E-03
22	0.23546265E-03
23	0.33678993E-03
24	0.44260229E-03
25	0.72434894E-03
26	0.11187989E-02
27	0.22375367E-02
28	0.44768206E-02
29	0.12673118E-01
30	0.40718615E-01
31	0.19765863E+00

```
      This program computes DFT directly.
      COMPLEX SUM(32),C,G
      DIMENSION Y(32)
      S=0.0
      PI=.31415927E1
      OPEN(UNIT=1,NAME='CELL.DAT',TYPE='OLD')
      DO 700 K=1,32
      SUM(K)=(0.,0.)
      L=K-1
      REWIND 1
        DO 600 N=1,32
        M=N-1
        I=L*M/32
        J=L*M-I*32
        B=FLOAT(J)*PI/16.
        G=(0.,0.)
        IF(J.EQ.0) G=(1.,0.)
        IF(J.EQ.8) G=(0.,1.)
        IF(J.EQ.16) G=(-1.,0.)
        IF(J.EQ.24) G=(0.,-1.)
        IF(CABS(G).EQ.0.) G=CMPLX(COS(B),-SIN(B))
        READ(1,500) C
500     FORMAT(1X,E15.8)
        SUM(K)=SUM(K)+C*G
600     CONTINUE
700   CONTINUE
      REWIND 1
      CLOSE(UNIT=1)
      DO 900 K=1,32
      Y(K)=(CABS(SUM(K)))**2
900   CONTINUE
      DO 1000 K=1,32
      S=S+Y(K)
1000  CONTINUE
      DO 2000 K=1,32
      Y(K)=Y(K)/S
2000  CONTINUE
      OPEN(UNIT=2,NAME='PSDBF.DAT',TYPE='OLD')
      DO 4000 K=1,32
      WRITE(2,3000) K-1,Y(K)
      WRITE(5,3000) K-1,Y(K)
3000  FORMAT(1X,I2,1X,E15.8)
4000  CONTINUE
      REWIND 2
      CLOSE(UNIT=2)
      STOP
      END
              !*********** PROGRAM END ***********!
```

CHAPTER IV

CONCLUSION

4.1 SUMMARY OF WORK DONE

The aim of this work was to develop computer programs in FORTRAN IV which could be used to conduct a preliminary investigation into some properties of radar clutter, based on the statistical evaluation of measured data. The data is to comprise of estimates of the samples of complex reflectivity of the source, as a function of both discrete range and discrete time, when illuminated by a stationary beam. Three properties were identified for computer-aided investigation, namely, probability distribution, correlation and power spectral density. Three distributional models (Rayleigh, Weibull and Lognormal) are commonly suggested for clutter data. It would be of interest to investigate the adequacy of these models. The correlation between the clutter variations as measured from two range cells which are separated spatially, is also of interest. Based on the assumption of wide-sense stationary, uncorrelated scattering (WSSUS), the scattering function description of a specific range cell is the PSD. The estimation of clutter PSD from the data is once again of interest. Three programs were developed during the course of this work:

<u>Program-I</u>:

With this program the clutter data can be tested for the adequacy of three types of probability distributions, namely the

Rayleigh, Weibull and Lognormal, which are commonly suggested as valid models. The method employed is a qualitative one. The resultant data is required to be plotted. The plot is a straight line if the particular model is adequate. The program processes clutter data pertaining to one range cell. An optional facility for time averaging the data initially is provided.

Program-II:

This program provides two methods for evaluating the cross correlation between the clutter returns obtained from two range cells:

a) Direct method

b) FFT method

The particular method is selected by the user. As a result of program design, in the direct method, the number of data points over which correlation is to be evaluated, the start-point in the first sequence and the time-delay in the second sequence are user-selected variable parameters, and the value of cross-covariance (normalised) is computed. In the FFT method on the other hand, the value of cross-correlation (normalised) is computed over the full length of the input sequences, for all values of the delay. In the second method, the sequences are assumed to be periodic. The FFT algorithm used is the same as in Program-III.

Program-III:

The program evaluates power spectral density (normalised) based on the DFT of the clutter data from one range cell. A radix 2 in-place decimation-in-time FFT algorithm is used in computing the DFT.

Constructed data was required, to test the programs. A program was, therefore, also written to generate Rayleigh, Weibull and Lognormal distributed numbers. Programs I, II and III were run with generated test data. They were found to carry out their processing operations as designed.

4.2 SOME SUGGESTED IMPROVEMENTS

A scheme for preprocessing the raw measured data, i.e. one that essentially re-arranges the data cell-wise, needs software implementation. It involves transposition of a matrix of large size (e.g. 1024x300 size, each element in turn consisting of two real numbers denoting a complex sample). (Section 1.2.3 refers).

The test for the adequacy of the distributional models is based on the visual assessment of a graphical plot and is qualitative in nature. Therefore, the inclusion of a second test, which is quantitative, is suggested. One such test is the chi-squared test [4] (Section 2.4 refers).

APPENDIX A : COMPUTER PROGRAMS

A.1 Program-I : An aid to the verification of Rayleigh, Weibull and Lognormal models for the clutter data from one range cell.

A.2 Program-II : Estimation of correlation between the clutter returns from two range cells.

A.3 Program-III : Estimation of Power Spectral Density of the clutter from one range cell.

A.4 A program for the generation of Rayleigh, Weibull and Lognormal distributed numbers.

A.5 List of data files required while using the programs.

****

PROGRAMS FOR STATISTICAL EVALUATION OF MEASURED CLUTTER DATA

M. Tech. Project, 1989, Jayaswal.R. EE Dept, IIT Kanpur.

PROGRAM - I : Adequacy of RAYLEIGH, WEIBULL and LOGNORMAL Models for the clutter data from one Range Cell - A Qualitative Assessment.

PROGRAM - II : CORRELATION between the clutter data from Two Range Cells.

PROGRAM - III : POWER SPECTRAL DENSITY of the clutter from one Range Cell based on DFT of the data.

****

```
*************************************************************************

                              PROGRAM - I
                              -----------

THIS PROGRAM ENABLES A QUALITATIVE ASSESSMENT OF THE ADEQUACY OF THE
FOLLOWING DISTRIBUTIONAL MODELS FOR THE CLUTTER DATA  :-

                 (A)          RAYLEIGH
                 (B)          WEIBULL
                 (C)          LOGNORMAL

*************************************************************************
      PROGRAM MODELS
      COMPLEX VAL
      DIMENSION AZ(1024),BZ(1024),CZ(1024),DZ(1024)
      COMMON AZ/AREA1/BZ/AREA2/CZ/AREA3/DZ
      WRITE(5,2)
      FORMAT(/////////15X,'ENTER THE NO. OF (COMPLEX) DATA POINTS'//
     1       15X,'TO BE OPERATED UPON FROM FILE CELL.DAT'//
     2       15X,'( NOT TO EXCEED 1024 )  : ',$)
      READ(5,3,ERR=1) NUM
      FORMAT(I4)
      IF((NUM.GT.1024).OR.(NUM.LT.1)) GOTO 1
      JJ=NUM
      OPEN(UNIT=1,NAME='CELL.DAT',TYPE='OLD')
      OPEN(UNIT=2,NAME='AVERAG.DAT',TYPE='OLD')
      DO 5 J=1,NUM
      READ(1,4) VAL
4     FORMAT(1X,E15.8)
      WRITE(2,4) VAL
5     CONTINUE
      REWIND 1
      REWIND 2
      CLOSE(UNIT=1)
      CLOSE(UNIT=2)
6     WRITE(5,7)
7     FORMAT(///36X,'MENU'/36X,'____'///)
      WRITE(5,10)
10    FORMAT(20X,'1',6X,'TEST FOR RAYLEIGH/WEIBULL MODEL'//
     1       20X,'2',6X,'TEST FOR LOGNORMAL MODEL'//
     2       20X,'3',6X,'AVERAGE THE DATA OVER GIVEN TIME PERIOD' /
     3       20X,'4',6X,'EXIT'///
     4       26X,'ENTER A NUMBER : ',$)
20    READ(5,30,ERR=40) I
30    FORMAT(I1)
      IF(I.GT.4) GOTO 40
      IF(I.EQ.3) CALL TIMEAV(NUM,JJ)
      IF(I.EQ.1) CALL WEITST(JJ)
      IF(I.EQ.2) CALL LNTEST(JJ)
      IF(I.EQ.4) STOP
      GOTO 6
40    WRITE(5,50)
50    FORMAT(//26X,'INCORRECT ENTRY. ENTER AGAIN : ',$)
      GOTO 20
      STOP 'END OF RUN'
      END
-------------------------------------------------------------------------
 ***  SUBR-1 : FINDS LINEAR TIME AVERAGE OF THE ORIGINAL DATA  ***
```

```
      SUBROUTINE TIMEAV(N,J)
      COMPLEX CC(1024),SUM,C
      WRITE(5,2)
      FORMAT(/16X,'L = NO. OF CONSECUTIVE DATA VALUES TO BE AVERAGED'
    1 //16X,'ENTER  L  :',$)
      READ(5,5) L
      FORMAT(I4)
      IF(L.GT.N) GOTO 90
      OPEN(UNIT=1,NAME='CELL.DAT',TYPE='OLD')
      J=0
      K=1
      LL=L
      LLL=L
 0    SUM=(0.,0.)
      J=J+1
      DO 30 I=1,LLL
      READ(1,20) C
 0    FORMAT(1X,E15.8)
      SUM=SUM+C
 0    CONTINUE
      CC(J)=SUM/FLOAT(LLL)
      IF(J.EQ.1024) GOTO 70
      K=K+L
      LL=LL+L
      IF(K.GT.N) GOTO 70
      IF(LL.GT.N) GOTO 60
      GOTO 10
 60   LLL=N+L-LL
      GOTO 10
 70   REWIND 1
      CLOSE(UNIT=1)
      OPEN(UNIT=2,NAME='AVERAG.DAT',TYPE='OLD')
      WRITE(2,20)(CC(I),I=1,J)
      REWIND 2
      CLOSE(UNIT=2)
      WRITE(5,80) J
 80   FORMAT(//21X,'AVERAGED DATA HAS ',I4,'  COMPLEX SAMPLES'//)
      GOTO 200
 90   WRITE(5,100) N
 100  FORMAT(24X,'L EXCEEDS TOTAL NO. OF DATA (=',I4,' )'//)
      GOTO 1
 200  RETURN
      END
-----------------------------------------------------------------------
  ***  SUBR-2 : CARRIES OUT STEPS TO CHECK THE ADEQUACY OF THE    ***
  ***             RAYLEIGH AND WEIBULL MODELS.                    ***
      SUBROUTINE WEITST(NO)
      COMPLEX B(1024)
      DIMENSION R1(1024),F1(1024),X(1024),Y(1024)
      COMMON R1/AREA1/F1/AREA2/X/AREA3/Y
      WRITE(5,5)
 5    FORMAT(25X,'WEIBULL DISTRIBUTION TEST RESULTS')
      WRITE(5,8)
 8    FORMAT(25X,'------- ------------ ---- -------')
      OPEN(UNIT=1,NAME='AVERAG.DAT',TYPE='OLD')
      READ(1,10)(B(I),I=1,NO)
 10   FORMAT(1X,E15.8)
      DO 20 I=1,NO
      R1(I)=CABS(B(I))
 20   CONTINUE
```

```
      WRITE(5,30)AME.VAR
      FORMAT(29X,'MEAN=',E17.8/29X,'VARIANCE=',E17.8/)
      CALL CUM(NO,NOE)
      DO 40 J=1,NOE
      Y(J)=ALOG(ALOG(1./(1.-F1(J))))
      CONTINUE
      WRITE(5,45)
      FORMAT(31X,'X',16X,'Y'/31X,'-',16X,'-')
      WRITE(5,50)(X(L),Y(L),L=1,NOE)
      FORMAT(23X.2E17.8)
      WRITE(5,52)
      FORMAT(/31X,'PLOT (X,Y) POINTS')
      WRITE(5,55)
 5    FORMAT(27X,'IF POINTS LIE ON A STR LINE')
      WRITE(5,57)
 7    FORMAT(26X,'DISTRIBUTION IS WEIBULL WITH'/29X,'ETA=SLOPE')
      CALL FIT(NO,NOE,SL,YIN)
      A=YIN/SL
      SIG=EXP(-A)
      WRITE(5;60)SIG
0     FORMAT(29X,'SIGMA=',E17.8/)
      WRITE(5,70)
0     FORMAT(29X,'BEST STR LINE FIT IS :')
      WRITE(5,80) SL,YIN
0     FORMAT(29X,'SLOPE=',E17.8/29X,'Y INTERCEPT=',E17.8/)
      REWIND 1
      CLOSE(UNIT=1)
      OPEN(UNIT=2,NAME='WBTEST.DAT',TYPE='OLD')
      WRITE(2,90)(X(I),Y(I),I=1,NOE)
0     FORMAT(1X,2E15.8)
      REWIND 2
      CLOSE(UNIT=2)
      RETURN
      END
------------------------------------------------------------------------
 ***  SUBR-3 : ARRANGES MAGNITUDE VALUES OF THE DATA IN ASCENDING   ***
 ***           ORDER.                                               ***
      SUBROUTINE ASC(N)
      DIMENSION R(1024)
      COMMON R
      M=N-1
      DO 20 K=1,M
           DO 10 L=K,M
           IF(R(K).LE.R(L+1)) GOTO 10
           T=R(K)
           R(K)=R(L+1)
           R(L+1)=T
)          CONTINUE
)     CONTINUE
      IF(R(N).NE.0.0) GOTO 40
      WRITE(5,30)
)     FORMAT(25X,'ALL DATA ARE ZERO'//25X,'STOPPING')
      STOP
)     RETURN
      END
------------------------------------------------------------------------
 ***  SUBR-4 : EVALUATES MEAN AND VARIANCE  ***
      SUBROUTINE MV(N,AM,V)
```

```
      P=0
      Q=0
      C=N
      DO 10 I=1,N
      P=P+R(I)
      CONTINUE
      AM=P/C
      DO 20 J=1,N
      Q=Q+(R(J)-AM)**2
      CONTINUE
      V=Q/C
      RETURN
      END
------------------------------------------------------------------------
**    SUBR-5 : FINDS THE VALUE OF THE CUMULATIVE DISTRIBUTION      ***
**             FUNCTION AT EACH DATA POINT.                        ***
      SUBROUTINE CUM(N,NS)
      DIMENSION R(1024),F(1024),EX(1024)
      COMMON R/AREA1/F/AREA2/EX
      IS=0
      NS=N
      DO 10 K=1,N
      IF(K.EQ.N) GOTO 20
               IF((R(K).NE.R(K+1)).AND.(R(K).NE.0.0)) GOTO 5
               NS=NS-1
               GOTO 10
               IS=IS+1
               F(IS)=FLOAT(K)/FLOAT(N)
               EX(IS)=ALOG(R(K))
      CONTINUE
      F(NS)=.999999
      EX(NS)=ALOG(R(N))
      RETURN
      END
------------------------------------------------------------------------
***   SUBR-6 : FITS BEST STRAIGHT LINE THROUGH (X,Y) POINTS      ***
***                      IN MAIN PROGRAM                          ***
      SUBROUTINE FIT(N,NS,S,YT)
      DIMENSION A(1024),B(1024)
      COMMON /AREA2/A/AREA3/B
      G=NS
      ASUM=0
      BSUM=0
      AASUM=0
      ABSUM=0
      DO 100 I=1,NS
      ASUM=ASUM+A(I)
      BSUM=BSUM+B(I)
      AASUM=AASUM+A(I)**2
      ABSUM=ABSUM+A(I)*B(I)
      CONTINUE
      D=G*AASUM-ASUM**2
      S=(G*ABSUM-ASUM*BSUM)/D
      YT=(AASUM*BSUM-ASUM*ABSUM)/D
      RETURN
      END
------------------------------------------------------------------------
**    SUBR-7 : CARRIES
```

```
      SUBROUTINE LNTEST(NO)
      COMPLEX B(1024)
      DIMENSION RR(1024),FF(1024),X(1024),Y(1024)
      COMMON RR/AREA1/FF/AREA2/X/AREA3/Y
      WRITE(5,5)
      FORMAT(23X,'LOGNORMAL DISTRIBUTION TEST RESULTS'/23X,
   1            '--------- ------------ ---- -------')
      OPEN(UNIT=1,NAME='AVERAG.DAT',TYPE='OLD')
      READ(1,10)(B(I),I=1,NO)
      FORMAT(1X,E15.8)
      DO 20 I=1,NO
      RR(I)=CABS(B(I))
)     CONTINUE
      CALL ASC(NO)
      CALL CUM(NO,NOE)
      DO 30 I=1,NO
       Y(I)=RR(I)
       RR(I)=ALOG(RR(I))
)     CONTINUE
      CALL MV(NO,AME,VAR)
      WRITE(5,40) AME,VAR
)     FORMAT(28X,'MEAN=',E16.8/28X,'VARIANCE=',E16.8/28X,
   1                   'OF LN OF DATA'/)
      DO 50 I=1,NO
       RR(I)=Y(I)
)     CONTINUE
      S=SQRT(VAR)
      WRITE(5,55)
5     FORMAT(35X,'X',16X,'Y'/35X,'-',16X,'-')
      DO 90 J=1,NOE
       X(J)=(X(J)-AME)/S
       Y(J)=FF(J)
       A=ABS(X(J))
       CALL ERF(A,E)
       IF(X(J)) 60,70,70
)       X(J)=.5-E
        GOTO 80
)       X(J)=.5+E
)     WRITE(5,85) J,X(J),Y(J)
5     FORMAT(18X,I4,4X,2E17.8)
)     CONTINUE
      WRITE(5,95)
5     FORMAT(/32X,'PLOT (X,Y) POINTS'/27X,'IF POINTS LIE ON A STR LINE
   1         /28X,'DISTRIBUTION IS LOGNORMAL'/)
      CALL FIT(NO,NOE,SLO,YI)
      WRITE(5,100) SLO,YI
00    FORMAT(30X,'BEST STR LINE FIT IS:'/
   1         30X,'SLOPE=',E16.8/27X,'Y INTERCEPT=',E16.8)
      PLOT (X(I),Y(I),I=1,NOE)
      PLOT Y=SLO*X+YI
      REWIND 1
      CLOSE(UNIT=1)
      OPEN(UNIT=2,NAME='LNTEST.DAT',TYPE='OLD')
      WRITE(2,200)(X(I),Y(I),I=1,NOE)
00    FORMAT(1X,2E15.8)
      REWIND 2
      CLOSE(UNIT=2)
      RETURN
```

```
*  SUBR-8 :COMPUTES THE VALUE OF THE ERROR FUNCTION  ***
   SUBROUTINE ERF(H,Q)
   DIMENSION U(16),W(16)
   F(X)=EXP(-X*X)
   DATA U(1),U(2),U(3),U(4)/.0483077,.1444720,.2392874,.3318686/
   DATA U(5),U(6),U(7),U(8)/.4213513,.5068999,.5877158,.6630443/
   DATA U(9),U(10),U(11),U(12)/.7321821,.7944838,.8493676,.8963212/
   DATA U(13),U(14),U(15),U(16)/.9349061,.9647623,.9856115,.9972639/
   DATA W(1),W(2),W(3),W(4)/.09654009,.09563872,.09384440,.09117388/
   DATA W(5),W(6),W(7),W(8)/.08765209,.08331192,.07819390,.07234579/
   DATA W(9),W(10),W(11)/.06582222,.05868409,.05099806/
   DATA W(12),W(13),W(14)/.04283590,.03427386,.02539207/
   DATA W(15),W(16)/.01627439,.00701861/
   Q=0
   IF(H.EQ.0.) GOTO 20
   A=H/(2.*SQRT(2.))
   DO 10 I=1,16
   X=A*U(I)
   X1=A+X
   X2=A-X
   Q=Q+W(I)*(F(X1)+F(X2))
   CONTINUE
   Q=Q*A*.5641896
   RETURN
   END   !************ END OF PROGRAM - I ***********!
```

A.2

```
*************************************************************************

                              PROGRAM - II
                              ------------

   THIS PROGRAM EVALUATES THE CORRELATION ( SPATIAL ) BETWEEN THE
   CLUTTER RETURNED FROM TWO RANGE CELLS. THE USER CAN SELECT EITHER
   OF TWO METHODS PROVIDED :

                 (A)            DIRECT        METHOD

                 (B)            FFT           METHOD

**************************************************************************
      PROGRAM CORREL
      COMPLEX SEQ(1024)
      COMMON /AREA1/SEQ
50    WRITE(5,100)
00    FORMAT(//////////////////////)
      WRITE(5,200)
00    FORMAT(///6X,'CHOOSE A METHOD :-'//)
      WRITE(5,300)
00    FORMAT(23X,'1        DIRECT'//
  1          23X,'2         FFT'///)
      WRITE(5,400)
.00   FORMAT(6X,'ENTER 1 OR 2'//
  1          6X,'( Any other character EXITS )'//
  2          6X,'ENTRY  :',$)
      READ(5,500,ERR=600) M
500   FORMAT(I2)
      IF((M.NE.1).AND.(M.NE.2)) GOTO 600
      WRITE(5.100)
      CALL NUMDAT(NUM)
      WRITE(5,100)
      IF(M.EQ.1) CALL CORDIR(NUM)
      IF(M.EQ.2) CALL CORFFT(NUM)
      GOTO 50
600   STOP 'END OF RUN'
      END
------------------------------------------------------------------   ----
  ***  SUBR-1: COMPUTES CORRELATION COEFFICIENT ( COVARIANCE, NO.  .ISED
  ***          BY THE DIRECT METHOD , USING THE VALUES OF 'NOS', 'ISTART
  ***          AND 'ITAU' SUPPLIED TO IT.
      SUBROUTINE CORDIR(NO)
      COMPLEX DAT,DATK(1024),DATKK(1024),UK,UKK,SUM,RO
      SUM=(0.,0.)
      WRITE(5,5)
5     FORMAT(////////////////31X,'C O R R E L A T I O N'/31X,
  1                             '---------------------'/)
      CALL TIMES (NO,NOS,ISTART,ITAU)
      OPEN(UNIT=1,NAME='ONESET.DAT',TYPE='OLD')
      IF(ISTART.EQ.1) GOTO 30
      M=ISTART-1
      DO 20 I=1,M
      READ(1.10) DAT
10    FORMAT(1X.E15.8)
20    CONTINUE
30    CONTINUE
      DO 40 J=1.NOS
```

```
      READ(1,10) DATK(J)
      SUM=SUM+DATK(J)
10    CONTINUE
      UKR=REAL(SUM)/FLOAT(NOS)
      UKI=AIMAG(SUM)/FLOAT(NOS)
      UK=CMPLX(UKR,UKI)
      DO 50 I=1,NOS
      DATK(I)=DATK(I)-UK
50    CONTINUE
      REWIND 1
      CLOSE(UNIT=1)
      SUM=(0.,0.)
      OPEN(UNIT=2,NAME='TWOSET.DAT',TYPE='OLD')
      IF((ISTART.EQ.1).AND.(ITAU.EQ.0)) GOTO 70
      M=ISTART+ITAU-1
      DO 60 I=1,M
      READ(2,10) DAT
60    CONTINUE
70    CONTINUE
      DO 80 J=1,NOS
      READ(2,10) DATKK(J)
      SUM=SUM+DATKK(J)
80    CONTINUE
      UKKR=REAL(SUM)/FLOAT(NOS)
      UKKI=AIMAG(SUM)/FLOAT(NOS)
      UKK=CMPLX(UKKR,UKKI)
      DO 90 I=1,NOS
      DATKK(I)=DATKK(I)-UKK
90    CONTINUE
      REWIND 2
      CLOSE(UNIT=2)
      SUM=(0.,0.)
      SUMKD=0
      SUMKKD=0
      DO 100 J=1,NOS
      SUMKD=SUMKD+(CABS(DATK(J)))**2
      SUMKKD=SUMKKD+(CABS(DATKK(J)))**2
100   CONTINUE
      DENOM=SQRT(SUMKD)*SQRT(SUMKKD)
      DO 200 I=1,NOS
      A=REAL(DATK(I))
      AA=REAL(DATKK(I))
      B=AIMAG(DATK(I))
      BB=-AIMAG(DATKK(I))
      C=A*AA-B*BB
      D=A*BB+AA*B
      SUM=SUM+CMPLX(C,D)
200   CONTINUE
      PI=4.*ATAN(1.)
      CF=180./PI
      ROR=REAL(SUM)/DENOM
      ROI=AIMAG(SUM)/DENOM
      ROMAG=CABS(SUM)/DENOM
      IF(ROR.EQ.0.0) ROPHAS=90.
      IF((ROR.EQ.0.0).AND.((ROI/ROMAG).LT.0.0)) ROPHAS=-ROPHAS
      IF(ROMAG.EQ.0.0) ROPHAS=0.  !In this case ROPHAS is not defined.
      IF(ROR.NE.0.0) ROPHAS=ATAN(ABS(ROI/ROR))*CF
      IF((ROR.NE.0.0).AND.((ROI/ROR).LT.0.0)) ROPHAS=-ROPHAS
      WRITE(5,300) ROR,ROI,ROMAG,ROPHAS
```

```
     1          26X,'--- ---------- ---------- -- -'//
     2          26X,'r  =  '.E15.8' + j ( '.E15.8.' )'//
     3          26X,'I r I  =  ',E15.8//
     4          26X,'< r  ( in deg. )  =  ',E15.8////)
      RETURN
      END
-----------------------------------------------------------------------
 ***  SUBR-2: ASKS THE USER TO ENTER THE ( INTEGER ) VALUES OF :      ***
 ***          (a)  SEQUENCE LENGTH TO BE CONSIDERED  ( "NOSAMP" ).  ***
 ***          (b)  STARTPOINT IN SEQUENCE-ONE  ( "MSTART" ).          ***
 ***          (c)  ( STARTPOINT IN SEQUENCE-TWO ) - (b) = "MTAU".    ***
 ***          IT FORMS PART OF DIRECT METHOD.                         ***
      SUBROUTINE TIMES(N,NOSAMP,MSTART,MTAU)
      WRITE(5,10)
0     FORMAT(//14X,'ENTER CORRELATION LENGTH (= NO. OF DATA POINTS
    1 OVER WHICH'/14X,'THE CORRELATION COEFFICIENT IS TO BE COMPUTED
    2 ) :'//14X,'CORRELATION LENGTH=',$)
      READ(5,20,ERR=25) NOSAMP
0     FORMAT(I4)
      IF(NOSAMP.GE.1) GOTO 40
5     WRITE(5,30)
0     FORMAT(//28X,'INCORRECT ENTRY :ENTER AGAIN')
      GOTO 5
0     IF(NOSAMP.LE.N) GOTO 60
      WRITE(5,50) N
0     FORMAT(//15X,'CORRELATION LENGTH EXCEEDS NO. OF INPUT DATA (=',
    1          I4,')')
      GOTO 5
0     WRITE(5,70)
0     FORMAT(//18X,'ENTER TAU (TIME-DELAY) AS THE NO. OF INTERVENING'
    1          /18X,'DATA POINTS :'//18X,'TAU=',$)
      READ(5,20,ERR=75) MTAU
      IF(MTAU.GE.0) GOTO 90
75    WRITE(5.80)
30    FORMAT(//30X,'INCORRECT ENTRY (TAU) :')
      GOTO 60
90    IF((MTAU+NOSAMP).LE.N) GOTO 200
      WRITE(5,100) N
100   FORMAT(//26X,'(TAU+CORRELATION LENGTH)  EXCEEDS'
    1          /26X,'INPUT DATA LENGTH (=',I4,')')
      GOTO 60
200   WRITE(5.300)
300   FORMAT(//17X,'THE CORRELATION CALCULATION CAN BEGIN ANYWHERE'
    1          /17X,'IN THE INPUT SEQUENCE. ENTER :')
325   WRITE(5,350)
350   FORMAT(/17X,'START AT DATA POINT NO.(e.g. 1, 5, etc.)=',$)
      READ(5,20,ERR=375) MSTART
      IF(MSTART.GE.1) GOTO 500
375   WRITE(5,400)
400   FORMAT(//17X,'INCORRECT ENTRY (STARTING DATA POINT NO.). ENTER :'
      GOTO 325
500   IF((MSTART+MTAU+NOSAMP-1).LE.N) GOTO 800
      WRITE(5,600)
600   FORMAT(//12X,'(STARTING POINT)+TAU+(CORRELATION LENGTH)-1')
      WRITE(5.700) N
700   FORMAT(12X.'EXCEEDS INPUT DATA LENGTH (=',I4,'). RE-ENTER :')
      GOTO 325
800   WRITE(5.900) MSTART,MTAU,NOSAMP
```

```
)      FORMAT(//////////////28X,'STARTING DATA POINT NO.=',I4/28X,
  1    'TAU=',I4/28X,'CORRELATION LENGTH=',I4////)
       RETURN
       END
------------------------------------------------------------------------
***    SUBR-3: ASKS THE USER TO ENTER THE NO. OF ( COMPLEX ) DATA    ***
***            POINTS STORED IN THE INPUT DATA FILES 'ONESET.DAT'   ***
***            AND 'TWOSET.DAT'.                                     ***
       SUBROUTINE NUMDAT(N)
       WRITE(5,2)
       FORMAT(/18X,'ENTER THE NO. OF (COMPLEX) DATA AS ONE OF THESE
  1:'//18X,'2   4   8   16   32   64   128   256   512   1024'//
  2    26X,'DATA LENGTH TO BE PROCESSED =',$)
       READ(5,3,ERR=5) N
       FORMAT(I4)
       NC=1
       DO 4 NCC=1,10
       NC=NC*2
       IF(N.EQ.NC) GOTO 7
       CONTINUE
       WRITE(5,6)
       FORMAT(/26X,'NO. INCORRECT: ENTER AGAIN')
       GOTO 1
       WRITE(5,8) N
       FORMAT(/27X,'DATA LENGTH=',I4,1X,'BEING PROCESSED'///)
       RETURN
       END
------------------------------------------------------------------------
***    SUBR-4: COMPUTES DFT OF A COMPLEX SEQUENCE USING A RADIX 2    ***
***            IN-PLACE DECIMATION-IN-TIME FFT ALGORITHM.            ***
       SUBROUTINE FFT(N)
       COMPLEX A(1024),T,U,W
       COMMON /AREA1/A
       C=.31415927E1
       DO 5 M=1,10
       IF(M.EQ.10) GOTO 10
       N1=2**M
       IF(N-N1) 5,10,5
       CONTINUE
0      IM=M
       NN=N-1
       J=1
       DO 50 I=1,NN
       IF(I.GE.J) GOTO 20
         T=A(J)
         A(J)=A(I)
         A(I)=T
0      K=N/2
0      IF(K.GE.J) GOTO 40
         J=J-K
         K=K/2
         GOTO 30
0      J=J+K
0      CONTINUE
       DO 80 L=1,IM
       U=(1.,0.)
       II=2**L
       I2=II/2
```

```
      IF(I2.GT.2) W=CMPLX(COS(B),-SIN(B))
      DO 70 JJ=1,I2
        DO 60 KK=JJ,N,II
        MM=KK+I2
        T=U*A(MM)
        A(MM)=A(KK)-T
        A(KK)=A(KK)+T
        CONTINUE
      U=U*W
      CONTINUE
      CONTINUE
      RETURN
      END
----------------------------------------------------------------------
**    SUBR-5: ASSUMES THE TWO INPUT DATA SEQUENCES TO BE PERIODIC     **
**            AND COMPUTES THE VALUE OF CORRELATION ( NORMALISED )    **
**            FOR ALL TIME SHIFTS BY THE FFT METHOD.                  **
      SUBROUTINE CORFFT(N)
      COMPLEX B(1024),C(1024),D(1024)
      COMMON /AREA1/D
      CON=45./ATAN(1.)
      SUMBB=0
      SUMCC=0
      CALL HEADFT
      OPEN(UNIT=1,NAME='ONESET.DAT',TYPE='OLD')
      READ(1,10)(D(I),I=1,N)
      FORMAT(1X,E15.8)
      CALL FFT(N)
      DO 20 I=1,N
      B(I)=CONJG(D(I))
      SUMBB=SUMBB+(CABS(B(I)))**2
      CONTINUE
      REWIND 1
      CLOSE(UNIT=1)
      OPEN(UNIT=2,NAME='TWOSET.DAT',TYPE='OLD')
      READ(2,10)(D(I),I=1,N)
      CALL FFT(N)
      DO 30 J=1,N
      C(J)=D(J)
      SUMCC=SUMCC+(CABS(C(J)))**2
      CONTINUE
      REWIND 2
      CLOSE(UNIT=2)
      DEN=SQRT(SUMBB)*SQRT(SUMCC)
      DO 40 I=1,N
      D(I)=(B(I)*C(I))/DEN
      CONTINUE
      CALL FFT(N)
      OPEN(UNIT=3,NAME='CORFFT.DAT',TYPE='OLD')
      DO 60 I=1,N
      L=I-1
      RE=REAL(D(I))
      AI=-AIMAG(D(I))
      ABSL=CABS(D(I))
      IF(RE.EQ.0.0) PH=90.
      IF((RE.EQ.0.0).AND.((AI/ABSL).LT.0.0)) PH=-PH
      IF(ABSL.EQ.0.0) PH=0.    !In this case PH is not defined.
```

```
      FORMAT(4X,I4,2X,4(2X,E15.8))
      WRITE(3,55) L,RE,AI,ABSL,PH
      FORMAT(1X,I4,4(1X,E15.8))
      CONTINUE
      REWIND 3
      CLOSE(UNIT=3)
      RETURN
      END
------------------------------------------------------------------------
*     SUBR-6 : DISPLAYS HEADINGS. IT FORMS PART OF THE FFT METHOD.   ***
      SUBROUTINE HEADFT
      WRITE(5,1)
      FORMAT(15X,'CELL-TO-CELL CROSS CORRELATION'/
1            15X,'------------------------------'//10X,
2            'r = (COMPLEX-VALUED) CORRELATION COEFFICIENT'//3X,
3'Time-delay',55X,'Phase(r)'/3X,
4'  Index   ',6X,'Re(r)',12X,'Im(r)',12X,'Abs(r)',9X,' in deg.'/3X,
5'----------',6X,'-----',12X,'-----',12X,'------',9X,'--------')
      RETURN
      END    !************ END OF PROGRAM - II ************!
```

```
 ****************************************************************C
                                                                  C
                          PROGRAM - III                           C
                          -------------                           C
                                                                  C
 THIS PROGRAM EVALUATES THE POWER SPECTRAL DENSITY OF THE CLUTTER C
 FROM ONE RANGE CELL, GIVEN THE TIME-DOMAIN SEQUENCE CONSISTING   C
 OF THE ESTIMATES OF THE COMPLEX REFLECTIVITY OF THAT RANGE CELL. C
                                                                  C
 ****************************************************************C
      PROGRAM PSD
      COMPLEX B(1024)
      DIMENSION BB(1024),DD(513),CUM(513)
      COMMON /AREA1/B
      SUM=0.0
      SUM2=0.0
      CALL NUMDAT(NUM)
      CALL HEADER
      OPEN(UNIT=1,NAME='CELL.DAT',TYPE='OLD')
      READ(1,100)(B(I),I=1,NUM)
 00   FORMAT(1X,E15.8)
      REWIND 1
      CLOSE(UNIT=1)
      CALL FFT(NUM)
      DO 200 J=1,NUM
      BB(J)=(CABS(B(J)))**2
      SUM=SUM+BB(J)
 )0   CONTINUE
      NNN=NUM/2+1
      DD(1)=2.*BB(1)
      DO 300 M=2,NNN
      IARG=NUM-M+2
      DD(M)=BB(M)+BB(IARG)
 00   CONTINUE
      SUM2=SUM+BB(1)+BB(NNN)
      CUM(1)=DD(1)
      DO 400 N=2,NNN
      JARG=N-1
      CUM(N)=DD(N)+CUM(JARG)
 100  CONTINUE
      OPEN(UNIT=2,NAME='PSD.DAT',TYPE='OLD')
      DO 700 L=1,NNN
      LLL=L-1
      WRITE(2,500) LLL,(BB(L)/SUM),(10.*ALOG10(BB(L)/SUM)),
     1              (10.*ALOG10(CUM(L)/SUM2))
 500  FORMAT(1X,I4,3(1X,E15.8))
      WRITE(5,600) LLL,(BB(L)/SUM),(10.*ALOG10(BB(L)/SUM)),
     1              (10.*ALOG10(CUM(L)/SUM2))
 500  FORMAT(16X,I4,3(2X,E15.8))
 '00  CONTINUE
      MMM=NNN+1
      DO 900 JK=MMM,NUM
      LLL=JK-1
      WRITE(2,750) LLL,(BB(JK)/SUM),(10.*ALOG10(BB(JK)/SUM))
 '50  FORMAT(1X,I4,2(1X,E15.8))
      WRITE(5,800) LLL,(BB(JK)/SUM),(10.*ALOG10(BB(JK)/SUM))
 00   FORMAT(16X,I4,2(2X,E15.8))
 00   CONTINUE
      REWIND 2
```

```
      STOP 'END OF RUN'
      END
C-----------------------------------------------------------------------
C***  SUBR-1: GENERATES HEADINGS ON SCREEN  ***
      SUBROUTINE HEADER
      WRITE(5,10)
10    FORMAT(//////////////////////)
      WRITE(5,20)
20    FORMAT(12X,'POWER SPECTRAL DENSITY OF THE CLUTTER FROM',
     1           ' ONE RANGE CELL'
     2      /12X,'----------------------------------------',
     3           '---------------'/)
      WRITE(5,30)
30    FORMAT(12X,'( THE PSD VALUES ARE EVALUATED AS FRACTIONS ',
     1           'OF THE TOTAL'
     2      /12X,'POWER RETURNED FROM THAT RANGE CELL )'///)
      WRITE(5,40)
40    FORMAT(12X,'FREQUENCY        POWER                              ',
     1           'CUMULATIVE'
     2      /12X,'  INDEX      (NORMALISED)        p in dB            ',
     3           'POWER (dB)')
      WRITE(5,50)
50    FORMAT(/12X,'   (m)             (p)                             ',
     1           '(ONE-SIDED)'
     2      /12X,'---------   ------------        -------             ',
     3           '------------')
      RETURN
      END
C-----------------------------------------------------------------------
C***  SUBR-2: ASKS THE USER FOR THE NO. OF ( COMPLEX ) DATA POINTS TO **
C***          BE OPERATED UPON FROM FILE CELL.DAT.                     **
      SUBROUTINE NUMDAT(N)
      WRITE(5,1)
1     FORMAT(18X,'ENTER THE NO. OF (COMPLEX) DATA AS ONE OF THESE :'//
     1 18X,'2   4   8   16   32   64   128   256   512   1024')
2     READ(5,3) N
3     FORMAT(I4)
      NC=1
      DO 4 NCC=1,10
      NC=NC*2
      IF(N.EQ.NC) GOTO 6
4     CONTINUE
      WRITE(5,5)
5     FORMAT(/28X,'NO. INCORRECT: ENTER AGAIN')
      GOTO 2
6     WRITE(5,7) N
7     FORMAT(/27X,'DATA LENGTH=',I4,1X,'BEING PROCESSED'///)
      RETURN
      END
C-----------------------------------------------------------------------
C***  SUBR-3: COMPUTES THE DFT OF A COMPLEX SEQUENCE USING A RADIX 2   **
C***          IN-PLACE DECIMATION-IN-TIME FFT ALGORITHM                **
      SUBROUTINE FFT(N)
      COMPLEX A(1024),T,U,W
      COMMON /AREA1/A
      C=.3141592721
      DO 5 M=1,10
```

```
      N1=2**M
      IF(N-N1) 5,10,5
      CONTINUE
      IM=M
      NN=N-1
      J=1
      DO 50 I=1,NN
      IF(I.GE.J) GOTO 20
        T=A(J)
        A(J)=A(I)
        A(I)=T
      K=N/2
      IF(K.GE.J) GOTO 40
        J=J-K
        K=K/2
        GOTO 30
)     J=J+K
)     CONTINUE
      DO 80 L=1,IM
      U=(1.,0.)
      II=2**L
      I2=II/2
      B=C/FLOAT(I2)
      IF(I2.EQ.1) W=(-1.,0.)
      IF(I2.EQ.2) W=(0.,-1.)
      IF(I2.GT.2) W=CMPLX(COS(B),-SIN(B))
      DO 70 JJ=1,I2
      RR=REAL(U)
      GG=AIMAG(U)
        DO 60 KK=JJ,N,II
        MM=KK+I2
        R=REAL(A(MM))
        G=AIMAG(A(MM))
        TR=R*RR-G*GG
        TG=R*GG+RR*G
        T=CMPLX(TR,TG)
        A(MM)=A(KK)-T
        A(KK)=A(KK)+T
0       CONTINUE
      R=REAL(W)
      G=AIMAG(W)
      TR=R*RR-G*GG
      TG=R*GG+RR*G
      U=U*W
      U=CMPLX(TR,TG)
0     CONTINUE
0     CONTINUE
      RETURN
      END   !********** END OF PROGRAM - III **********!
```

```
*******************************************************************************C
                                                                               C
                    PROGRAM FOR TEST DATA SIMULATION                           C
                    --------------------------------                           C
                                                                               C
          THIS  PROGRAM  GENERATES :                                           C
                                                                               C
                    (A)          WEIBULL                                       C
                                                                               C
                    (B)          LOGNORMAL                                     C
                                                                               C
          DISTRIBUTED REAL NUMBERS ARRANGED IN ASCENDING ORDER                 C
          OF MAGNITUDE.                                                        C
                                                                               C
*******************************************************************************C
      PROGRAM RANDOM
      WRITE(5,2)
      FORMAT(//////////////////////)
      WRITE(5,3)
      FORMAT(31X,'NUMBER GENERATOR'/31X,
     1           '----------------')
      WRITE(5,4)
      FORMAT(////22X,'1      WEIBULL DISTRIBUTED NUMBERS'//22X,
     1              '2      LOGNORMAL DISTRIBUTED NUMBERS'////22X,
     2              'ENTER 1 OR 2 ( ANY OTHER NO. EXITS ) :',$)
      READ(5,5,ERR=6) I
      FORMAT(I1)
      IF(I.EQ.1) CALL WBL
      IF(I.EQ.2) CALL LOGNML
      IF((I.NE.1).AND.(I.NE.2)) GOTO 6
      GOTO 1
6     STOP 'END OF RUN'
      END
--------------------------------------------------------------------------------
***   SUBR-1: ASKS THE USER TO ENTER THE VALUES OF THE DISTRIBUTION       ***
***           PARAMETERS 'ETA' AND 'SIGMA' DESIRED, AND GENERATES         ***
***           REAL NUMBERS, IN ASCENDING ORDER OF MAGNITUDE, WITH A       ***
***           WEIBULL/RAYLEIGH DISTRIBUTION.                              ***
      SUBROUTINE WBL
      DIMENSION F(1024)
      CALL NUMDAT(N)
      WRITE(5,1)
      FORMAT(10X,'ENTER WEIBULL PARAMETERS ETA, SIGMA.'/
     1       10X,'( ETA = 2.0  FOR RAYLEIGH )'//
     2       10X,'ETA=',$)
      READ(5,2) ETA
      FORMAT(E15.8)
      WRITE(5,3)
      FORMAT(10X,'SIGMA=',$)
      READ(5,2) SIGMA
      FORMAT(1X,E15.8)
      FORMAT(27X,I4,3X,E15.8)
      M=N-1
      DO 10 I=1,M
      F(I)=FLOAT(I)/FLOAT(N)
      CONTINUE
```

```
      T=1./(1.-F(I))
      U=ALOG(T)
      V=ALOG(U)/ETA
      W=V+ALOG(SIGMA)
      X=EXP(W)
      WRITE(5,5) I,X
      WRITE(1,4) X
0     CONTINUE
      REWIND 1
      CLOSE(UNIT=1)
      RETURN
      END
C-----------------------------------------------------------------------------
C***  SUBR-2: ASKS HOW MANY NUMBERS ARE TO BE GENERATED FROM THE USER ***
      SUBROUTINE NUMDAT(NUM)
      WRITE(5,1)
      FORMAT(16X,'HOW MANY NUMBERS ARE TO BE GENERATED ?'//
     1          16X,'ENTER IT ( SHOULD NOT EXCEED 1024 ) : ',$)
      READ(5,3,ERR=4) NUM
      FORMAT(I4)
      IF((NUM.LT.1).OR.(NUM.GT.1024)) GOTO 4
      GOTO 6
      WRITE(5,5)
      FORMAT(/26X,'NO. INCORRECT: ENTER AGAIN')
      GOTO 2
      WRITE(5,7) NUM
      FORMAT(/24X,'DATA LENGTH=',I4,1X,'BEING PROCESSED'///////)
      RETURN
      END
C-----------------------------------------------------------------------------
C***  SUBR-3: COMPUTES THE VALUE OF THE ERROR FUNCTION  ***
      SUBROUTINE ERF(H,Q)
      DOUBLE PRECISION X1,X2,Z
      DIMENSION U(16),W(16)
      F(Z)=DEXP(-Z*Z)
      DATA U(1),U(2),U(3),U(4)/.0483077,.1444720,.2392874,.3318686/
      DATA U(5),U(6),U(7),U(8)/.4213513,.5068999,.5877158,.6630443/
      DATA U(9),U(10),U(11),U(12)/.7321821,.7944838,.8493676,.8963212/
      DATA U(13),U(14),U(15),U(16)/.9349061,.9647623,.9856115,.9972639/
      DATA W(1),W(2),W(3),W(4)/.09654009,.09563872,.09384440,.09117388/
      DATA W(5),W(6),W(7),W(8)/.08765209,.08331192,.07819390,.07234579/
      DATA W(9),W(10),W(11)/.06582222,.05868409,.05099806/
      DATA W(12),W(13),W(14)/.04283590,.03427386,.02539207/
      DATA W(15),W(16)/.01627439,.00701861/
      Q=0
      IF(H.EQ.0.) GOTO 30
      A=ABS(H)/(2.*SQRT(2.))
      DO 10 I=1,16
      X=A*U(I)
      X1=A+X
      X2=A-X
      Q=Q+W(I)*(F(X1)+F(X2))
      CONTINUE
      Q=Q*A
      IF(H) 20,30,30
      Q=-Q
      RETURN
      END
```

```
SUBR-4: ASKS USER FOR THE PERMISSIBLE ERROR  ( ABSOLUTE VALUE ) ***
        IN THE ( GENERATED ) LOGNORMAL-DISTRIBUTED NUMBERS AND  ***
        THE VALUES OF THE DISTRIBUTION PARAMETERS  'MU' AND     ***
        'SIGMA' DESIRED.                                        ***
SUBROUTINE PERM(ERR,UU,SIGMA)
WRITE(5,22)
FORMAT(15X,'ENTER THE VALUE OF ABSOLUTE ERROR PERMISSIBLE'/
      15X,'IN THE COMPUTATION AS  _.________  :')
READ(5,24,ERR=20) ERR
FORMAT(F10.7)
WRITE(5,26)
FORMAT(/////21X,'ENTER LOGNORMAL DISTRIBUTION PARAMETERS'/
             21X,'MU AND SIGMA AS  --.---  :'//
              1X,'MU=')
READ(5,28,ERR=25) UU
FORMAT(F7.3)
WRITE(5,30)
FORMAT(/1X,'SIGMA=')
READ(5,32,ERR=29) SIGMA
FORMAT(F7.3)
WRITE(5,34)
FORMAT(///////////35X,'GENERATED DATA'/35X,
                      '--------------'//)
RETURN
END
--------------------------------------------------------------------
SUBR-5: GENERATES REAL NMBERS IN ASCENDING ORDER OF MAGNITUDE  ***
        WITH A LOGNORMAL DISTRIBUTION.                         ***
SUBROUTINE LOGNML
CALL NUMDAT(NO)
P=NO
CALL PERM(PE,U,SIG)
PE=PE*1.7724538
OPEN(UNIT=1,NAME='LOG.DAT',TYPE='OLD')
X=1.7724538/P-.8862269
CALL ERF(X,T)
DO 90 I=1,NO
F=(FLOAT(I)/P)*1.7724538
IF(I.EQ.(NO/2)) GOTO 30
IF(I.EQ.NO) GOTO 40
Y=.8862269+T-F
IF(ABS(Y).LE.PE) GOTO 50
C=.1
YO=Y
(O=X
TO=T
(=XO-C*YO/ABS(YO)
CALL ERF(X,T)
=.8862269+T-F
F(ABS(Y).LE.PE) GOTO 50
F(ABS(Y).LT.ABS(YO)) GOTO 10
=XO
=TO
=.1*C
F(C.LT.(1.E-10)) GOTO 50
OTO 20
=0.
```

```
      I=0.
      GOTO 50
40    X=4.534099
50    Z=EXP(X*SIG+U)
60    WRITE(5,70) I,Z
70    FORMAT(27X,I4,3X,E15.8)
      WRITE(1,80) Z
80    FORMAT(1X,E15.8)
90    CONTINUE
      REWIND 1
      CLOSE(UNIT=1)
      RETURN
      END   !***** END OF TEST DATA SIMULATION PROGRAM *****!
```

A.5 List of data files required while using the programs.

All input, output and intermediate data files used in any of the programs are required to be created before run-time. The maximum length of data which the programs are designed to handle is a one-dimensional array of 1024 (complex) sample values. Program-wise list of data files is given below:

<u>Program-I</u>

Program Name : PROGRAM MODELS

Input data file

File name : CELL.DAT

Nature of data: Complex reflectivity samples of one range cell

Type of data : complex

Data format : 1X, E15.8/1X,E15.8

(Each complex sample is stored as two successive real numbers, being its in-phase and quadrature components, to be immediately followed by the next complex sample in the sequence).

Data entry mode : by the user

Intermediate storage file:

File-name : AVERAG.DAT

Nature of data: Time-averaged values of input data

Type of data : complex

Data format : 1X,E15.8/1X,E15.8

Data entry mode : by program

(Time-averaging is an optional facility provided in the program)

Output data files:

a) File-name : WBTEST.DAT

Nature of data: Set of (x,y) points to be plotted as a result of the Weibull/Rayleigh test program segment processing. (Each record in the output consists of two real numbers corresponding to the values of X and Y respectively).

Data format : 1X, 2E15.8

Data entry mode: by program

b) File-name : LNTEST.DAT

Nature of data : Set of (x,y) points to be plotted as a result of the Lognormal test program segment processing. (Each record in the output consists of two real numbers corresponding to the values of X and Y respectively).

Data format : 1X, 2E15.8

Data entry mode: by program

<u>Program-II</u>

Program name : PROGRAM CORREL

Input data files:

a) File-name : ONESET.DAT

Nature of data : Complex reflectivity samples of the first range cell.

Type of data : Complex

Data format : 1X, E15.8/1X,E15.8

Data entry mode : by the user.

b) File-name : TWOSET.DAT

Nature of data : Complex reflectivity samples of the second range cell.

Type of data : Complex

Data format : 1X, E15.8/1X, E15.8

Data entry mode : by the user

Intermediate storage files : none

Output data file (for FFT method only):

File-name : CORFFT.DAT

Nature of data : Set of normalised correlation values, corresponding to all values of time-delay by the FFT method. The exact output consists of a set of 5 real numbers per record, corresponding to

m; Re(r);Im(r); Abs(r); Ph(r) (deg)

where

m is the time delay index

r is the (complex-valued) correlation (normalised) and

Re(.), Im(.), Abs(.) and Ph(.) denote respectively its real part, imaginary part, magnitude and phase.

Data format : 1X, I4, 4(1X, E15.8)

Data entry mode : by program.

Note: Output in case of the direct method is a single value of normalised cross covariance, which is displayed on the VDU, and no output data file is required.

Program-III

Program name : PROGRAM PSD

Input data files:

File-name : CELL.DAT

Nature of data : Complex reflectivity samples from one range cell.

Type of data : Complex

Data format : 1X, E15.8/1X, E15.8

Data entry mode : by the user

Intermediate storage file : none

Output data file:

File-name : PSD.DAT

Nature of data : Normalised PSD values corresponding to all values of the frequency index (k). The exact output consists of four real numbers per record corresponding to k; PSD (normalised) (p); p(in dB); cumulative value of PSD (one-sided) (in dB).

Data format : 1X, I4, 3(1X, E15.8)

Data entry mode : by program

Program for the generation of Rayleigh, Weibull and Lognormal distributed numbers:

Program name : PROGRAM RANDOM

Input data files : none

Intermediate storage files : none

Output data files:

a) File-name : WBL.DAT

Nature of data : Set of real numbers arranged in ascending order of magnitude, which satisfy a Rayleigh or Weibull distribution, depending upon the parameter values entered by the user.

Data format : 1X, E15.8

Data entry mode : by program

b) File-name : LOG.DAT

Nature of data : Set of real numbers arranged in ascending order of magnitude, which satisfy a Lognormal distribution, in accordance with the parameter values entered by the user.

Data format : 1X, E15.8

Data entry mode : by program

Note 1: Output data files (a) and (b) are required for storing, respectively, the generated (a) Weibull/Rayleigh and (b) Lognormal distributed numbers. Only one of these need be created, if data is to be generated pertaining to the concerned distribution only.

2. Two further processing steps are required before the data as generated above can be used as test data for Program-I, viz. (a) jumbling the generated numbers in pseudo-random fashion, (b) creating complex numbers, whose magnitude values are equal to the numbers in (a) (e.g. by assigning a phase value to each in pseudo-random fashion).

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